

BEMACS Bocconi
30401 MATHEMATICS AND STATISTICS - MODULE 2 (STATISTICS)
A.A. 2019/20
EXERCISES ON CONDITIONAL PROBABILITY AND INDEPENDENCE

- (1) A parallel system is composed of three sub circuits, two formed of one single relay and one of two relays in series. All relays function independently with probability of being closed p_1, p_2, p_3, p_4 , respectively. Compute the probability the current flows through the system. Recompute if $p_1 = p_2 = p_3 = p_4 = p$ and then choose $p = 1/2$.

[Answer: $1 - (1 - p_1)(1 - p_2)(1 - p_3p_4)$; $2p - 2p^3 + p^4$; 0.8125]

- (2) Certain items are produced by three manufacturers A, B, C in the following percentages 20%, 30%, 50% respectively. The quality of an item produced by A is not acceptable with probability 0.01, if produced by B is 0.02 and 0.05 if it comes from C.

a) Compute the probability that, chosen an item at random, its quality is not acceptable.

b) If its quality is not acceptable, what is the probability the item has been produced by A?

[Answer: 33/1000; 2/33]

- (3) There are three coins in a box. One is two-headed coin, another is a fair coin and the third is two tailed-coin. One coin is chosen at random and then flipped. What is the probability the flip lands on head? If it lands on head, which is the probability the other face is head?

[Answer: 1/2; 2/3]

- (4) An urn contains seven white and three black balls. Three balls are chosen without replacement. What is the probability to choose three white balls? What is the probability the third ball is white?

[Answer: 7/24; 7/10]

- (5) MicroBur company sells office furniture and computers. A firm wants to renew its belongings with two different contracts, one for the furniture and another for the computers. MicroBur believes there is 60% probability to sell the furniture and, if it does, a probability of 2/3 to get also the contract for the computers signed. While if the furniture is not sold, the probability the computers will be sold is 30%.

a) Compute the probability that

- i) MicroBur will get the two contracts signed,
- ii) MicroBur will get only the contract related to the computers,
- iii) MicroBur will get the contract related to the computers,
- iv) MicroBur will get at least one of the two contracts,
- v) MicroBur will get only one of the two contracts.

b) MicroBur knows it has sold the computers, what is the probability it will sell also the furniture?

[Answer: a) 40%; 12%; 52%; 72%; 32%; b) $10/13 \simeq 0.769$]

- (6) Urn A contains two white balls and two black balls; urn B contains three white and two black balls. A ball is selected from A and placed into urn B. Then, a ball is selected from B and the ball is white. Which is the probability that the first ball selected (from A) was also white?

[Answer: $4/7$]

- (7) An exams consists of 5 multiple choice tests with 4 answers (only one of which is correct). By choosing the answer among the 4 randomly, compute a) the probability to answer correctly at least to four question; b) the probability to answer correctly to none; c) the probability to answer correctly at least to one question.

[Answer: $1/4^3$; $\frac{3^5}{4}$; $1 - \frac{3^5}{4}$]

- (8) From a bunch of n keys, we look for the one which opens the door of our office (we try one key at a time and we discard the once already tested). What is the probability that we find the right one after exactly k trials ($k \leq n$)?

[Answer: $1/n$]

- (9) From a bunch of twenty keys, we look for the right one testing one by one at random. Compute the probability that we find the right one after exactly four trials

- a) if we discard the keys tested;
- b) we mix up the tested keys with the others.

[Answer: $1/20$; 0.04287]

- (10) A system S consists of two subsystems S_1 and S_2 in series. The subsystem S_1 (resp., S_2) is formed of two components a and b (resp., c and d) in parallel, whose probabilities of being functional are respectively p_a and p_b (resp., p_c and p_d).

- a) compute the probability the system functions;
- b) compute the probability the system functions if d is not working;
- c) compute the probability d is not working is the system does.

[Answer: $(p_a + p_b - p_a p_b)(p_c + p_d - p_c p_d)$; $(p_a + p_b - p_a p_b)p_c$; $\frac{p_c(1-p_d)}{p_c+p_d-p_c p_d}$]

- (11) Suppose an ordinary deck of fifty-two cards is shuffled and the four players are dealt thirteen cards each. What is the probability that each player receives one ace?

[Answer: 0.105]

- (12) An item is produced by two companies A and B in the same percentages. It has been estimated that, a randomly chosen failed item is produced by A with probability 0.25. If items from A are defective with probability 5%, what is the probability of defective items from the company B?

[Answer: 15%]

- (13) There are two urns: the first contains two black and five white balls, while the second three black and two white. First a urn is chosen at random, a ball is selected and inserted into the other urn. Finally another ball is selected from the second urn. What is the probability that two white balls have been withdrawn?

[Answer: $23/70$]

- (14) Shooting at a target, one has the probability of 20% of hitting the target. If shooting twice, what is the probability of hitting it? (assume the shots are independent).
[Answer: 36%]
- (15) John sent his application to both Oxford and Stanford. The probability of being admitted to Oxford is 0.4, to Stanford is 0.5, to both is 0.2. What is the probability that John application is rejected by both? What is the probability that John will be admitted to Stanford given he has been admitted to Oxford?
[Answer: 30%; 50%]
- (16) A prisoner is closed in a cell that has three doors: A, B, C. A leads the prisoner again to the cell after two hour trail, B after four hours, C immediately leads him to the exit. After rolling two dice at random the prisoner chooses:
- A if the sum is at most 5
- B if the sum is at least 10
- C otherwise.
If back to the cell, he chooses between the two remaining doors at random. What is the probability the prisoner takes at most four hours to exit?
[Answer: 7/9]
- (17) A factory receives some transistors from two other firms respectively in the proportions 0.6 from the first and 0.4 from the second. 95% of the transistors produced by the first and 80% of the ones produced by the second pass the quality test. Choosing a transistor at random and known it fails the test, compute the probability it has been produced by the second firm.
[Answer: 0.727]
- (18) 98% of all babies survive delivery. However, 20% of all births involve Cesarean sections and, when a Cesarean section is performed, the baby survives 96% of the time. What is the probability that a baby born without Cesarean section survives?
[Answer: 98.5%]
- (19) In a clinical test for a certain disease, 6% are positive to the test but only the 80% of those have the disease. Compute the probability that a person with the disease is positive to the test, if 5% is the proportion of people with the disease.
[Answer: 0.96]
- (20) A box contains thirty batteries, of which three are defective. Two batteries are chosen at random and without replacement from the box. Compute the probability that
a) both batteries are defective;
b) none of them is defective;
c) at least one is defective.
d) Repeat the exercise in the case we replace the first battery in the box before the second extraction.
[Answer: a) 0.00689 b) 0.8069 c) 0.1931 d) 0.01; 0.81; 0.19]

- (21) Two urns U_1 and U_2 contain three white and two black balls and two white and three black balls, respectively. We flip a coin such that the probability that it lands on head is 0.4. If head occurs, we withdraw with replacement from U_1 , otherwise from U_2 . Let W_i the event “white ball at the i -th withdrawal”. Compute:
- $\mathbb{P}(W_i)$ with $i = 1, 2, 3$;
 - $\mathbb{P}(W_3|W_1 \cap W_2)$;
 - $\mathbb{P}(U_1|W_1 \cap W_2)$. Recompute the probabilities in the case without replacement.
- [Answer: a) Any 12/25; b) 13/25; c) 3/5; without replacement to be computed]

- (22) Two cards are selected at random from a deck of fifty-two. Compute the probability:
- two aces are selected;
 - the second is an ace given the first was an ace;
 - the second is an ace.

[Answer: 1/221; 1/17; 1/13]

- (23) Workers from a French company are offered to attend a nationally certified training course. It is known that in the company, the probability that a worker who have attended the training course, improves the income within five years is 0.86. The probability for whom have not attended the course is 0.35. The 80% of workers who recently joined the company attend the course. What is the probability that a worker recently hired will increase his salary within five years? and if she/he does, which is the probability she/he attended the course?

[Answer: 75.80%; 90.76%]

- (24) An urn contains a white (W) and b black balls (B). One ball is randomly chosen from the urn and, then, it is replaced in the urn together two other balls of the same color and three balls of different color. Compute the probability that in four withdrawals one gets BWWB.

[Answer: $\frac{b+8}{a+b+15} \frac{a+5}{a+b+10} \frac{a+3}{a+b+5} \frac{b}{a+b}$]

- (25) An insurance company has an equal number of policyholders from the class A, e.g., “new policyholders”, and class B (and “old policyholders”). Suppose each year the probability that a person from the class A has an accident is a (respectively from B is b), independently each year. Assume a policyholder is randomly chosen and compute:

- the probability he/she will have an accident this year;
- the probability that he/she has an accident in two consecutive years;
- the probability that an accident is from a person belonging to class B;
- compute the probability that a person which had an accident in the first year has higher probability to have an accident in the second year with respect to a person who did not have the accident during the first year.

[Answer: $\frac{1}{2}(a+b)$; $\frac{1}{2}(a^2+b^2)$; $\frac{b}{a+b}$; $P(I_2|I_1) \geq P(I_2|I_1^c)$]

- (26) A quality check process consists of two independent quality tests ($k = 1, 2$). The k -th test rejects (R_k) a standard quality item (S) with probability $\mathbb{P}(R_k|S) = r_k$ while a defective item (D) is not rejected (A_k) with probability $\mathbb{P}(A_k|D) = a_k$. An item is considered good (A) if it is not rejected in any test. Conditionally to the

quality (S or D) the events A_1 and A_2 are independent, and the same holds for R_1 and R_2 . Compute the probability of:

- a) A standard quality item is rejected.
- b) A defective item is accepted.
- c) Consider $r_1 = 2\%$, $r_2 = 1\%$, $b_1 = 15\%$, $b_2 = 10\%$.

[Answer: $r_1 + r_2 - r_1 r_2$; $a_1 a_2$; 1%, 1.5%]

- (27) Consider a particle starting at 0 that at each step is equally likely to move one position forward (+1) or backward (-1).

- a) Describe the position of the particle after two steps;
- b) what is the probability that after 50 steps the particle is at 25? and at 26?
- c) what is the probability that the particle moved the first step forward given that at the step three is in -1?

[Answer: $p(-2) = p(2) = 1/4$, $p(0) = 1/2$; 0; $\binom{50}{38} \frac{1}{2^{50}}$; 1/3]