BEMACS Bocconi

30401 MATHEMATICS AND STATISTICS - MODULE 2 (STATISTICS)

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EXERCISES ON CONTINUOUS RANDOM VARIABLES

(1) Bus 33 arrives at Politecnico bus stop in average every 10 minutes. Let suppose that the waiting time for the bus is a uniform random variable X.
   (a) Determine the density function of X.
   (b) I've been waiting the 33 at the bus stop for 10 minutes and the bus has not arrived yet. What is the probability that it will arrive within the next 5 minutes?
   (c) Suppose the waiting time was exponential. Answer again to a) and b).

   $\big($Answer: a) $X \sim U(0, 20)$; b) 1/2; c) $X \sim Exp\,(10^{-1})$; $1 - e^{-1/2}$ $\big)$

(2) The power W dissipated by a resistor is proportional to the potential difference at the extremal points, i.e. $W = rV^2$ where r is a constant. Suppose $r = 2$ and V a normal random variable of mean 8 and standard deviation 1, compute:
   (a) $E(W)$;
   (b) $P(W > 200)$.

   $\big($Answer: a) 130 b) 0.023 $\big)$

(3) The length of an object is a random variable Y that can be thought as $Y = \mu + X$, where μ is the true unknown value of the length, while X is the random variable representing the error. We suppose X to be a normal random variable with mean 0 and variance σ^2. Moreover, we know that the probability that the measurement deviates more than 1 in absolute value from the true measure μ is 0.05.
   (a) Determine σ.
   (b) Compute the probability that out of 100 measurements at least 3 deviates more than 1 in absolute value from μ.

   $\big($Answer: a) 0.5102 b) 0.8817 $\big)$

(4) (Inverse transformation method) Let X be a random variable with continuous distribution function $F_X(x)$. It is not difficult to show that $Y = F_X(X)$ is uniform on [0,1]. Show this result if $X \sim Exp(1)$.

(5) A department store receives the products from two different suppliers F_1 e F_2 in the percentages 70% and 30%, respectively. A certain product is said to meet the technical standards if a certain characteristic of the product is between 29.6 and 30.4. We suppose that, for both providers, the characteristic of the product has a normal distribution with $\mu = 29.9$, $\sigma^2 = 0.09$ for supplier F_1 and $\mu = 30$, $\sigma^2 = 0.12$ for F_2. Find the percentage of the products sold by the department store which does not meet the technical standards.

   $\big($Answer: 0.2194 $\big)$

(6) A company produces tomato sauce cans. The exact weight of a can has to be within 485 and 515 grams. Suppose that cans weights are normally distributed with mean 495.5 grams and standard deviation 6.45 grams.
   (a) Compute the probability that a can weights more than 515 grams.
   (b) Compute the probability that a can, which meets the standards, weights more than 495.5 grams.
   (c) If the mean was 500.24 grams, compute the standard deviation in order to have that the percentage of cans whose weight is bigger than 515 grams is less or equal to 2.5%.

   (Answer: 1) 0.0013; 2) 0.526; 3) 7.53)

(7) The length of the wheel spokes of a certain bike produced by Rag s.p.a. is approximatively normal with mean 28.5 cm.
   (a) Find the percentage of spokes whose length differs in absolute value from the mean more than 2 times the standard deviation.
   (b) Consider 100 wheels, what is the probability that at least 3 have the length which differs in absolute value from the mean more than 2 times the standard deviation?

   (Answer: 1) 0.0456; 2) 0.8395)

(8) Suppose the height (in cm) X of 16-17 years old girls is normally distributed with parameters μ and σ^2. Moreover it is known that $\mathbb{P}(X \leq 160) = 0.5$ and $\mathbb{P}(X \leq 140) = 0.25$.
   (a) Find μ and σ.
   (b) Compute $\mathbb{P}(X \leq 190)$.
   (c) Compute the percentage of girls, whose height is bigger than 190 cm, which are more than 200 cm tall.

   (Answer: a) $\mu = 160$, $\sigma = 29.63$; b) 0.8438; c) 0.567)

(9) A certain kind of valves, produced in series, have the external diameter which has a normal distribution with mean $\mu = 12.50$ mm and standard deviation $\sigma = 0.020$ mm. According to the regulations standards the diameter should be in the interval (12.46, 12.54) (mm).
   (a) Find the probability that a valve meets the standards.
   (b) Find the probability that a valve, which meets the standards, has the diameter less than 12.52 mm.

   (Answer: 1) 0.9545; 2) 0.8576)

(10) The lifetime of certain components is an exponential random variable with parameter $\lambda = 5(10)^{-3}$.
   (a) Compute the probability that a component lasts for at least 200 hours.
   (b) Compute the probability that a component lasts at least 300 hours given that it has been functioning for 100 hours already.
   (c) Out of 50 components, which is the probability that only 3 of them last for more than 200 hours?

   (Answer: a) 0.3678 b) 0.368 c) $4.236\ (10)^{-7}$)

(11) A chemical company produces drugs. The production process includes to perform some chemical reactions, whose duration can be modeled by an exponential random variable T with mean 10^4 seconds.
  (a) Compute the probability the chemical reaction lasts for more than 10^2 seconds.
  (b) Compute the probability that a chemical reaction started 10^2 seconds ago lasts for less than 10^4 seconds.
  (c) If a reaction lasts less than 10^2 seconds, it is not enough and the reaction has to be performed again. In average after how many reactions we expect to perform before we need to repeat one?

$\big($Answer: a) 0.99 b) 0.6284 c)100 $\big)$

(12) The current I owing through a semiconductor diode can be estimated by using the Shockley equation $I = I_0 \left(e^{aV} - 1\right)$, where V is the voltage across the terminals of the diode, I_0 the reverse bias saturation current and a is a constant. Find the $\mathbb{E}(I)$ if $a = 5$, $I_0 = 10^{-6}$ and V is uniform on the interval (1,3).

$\big($Answer: 0.3269 $\big)$

(13) The time interval between two visits at a web site is an exponential random variable with parameter $\lambda = 3$ (minutes).
  (a) Compute the probability that for at least one minute there are no new accesses at the web site.
  (b) If in 2 minutes nobody visited the web site, what is the probability that in the next minute the web site will be visited?

$\big($Answer: a) 0.0498; b) 0.9502 $\big)$

(14) The duration of a component (in hours) has the exponential law with mean equal to 2 hours. What is the probability that out of 100 components 30 last more than 3 hours?
$\big($Answer: 0.0177 $\big)$

(15) We use batteries of two different brands in the same percentage. The ones produced by brand A last less than t with probability $1 - e^{-t}$, while for the ones produced by B this probability is $1 - e^{-2t}$. If one battery has lifetime in the interval $[1; 2]$, what is the probability it has been produced by A?
$\big($Answer: 0.6652 $\big)$

(16) The lifetime of a flashlight is a random variable T with density function $f(t) = c/t^3$ if $t > 100$ and null otherwise.
  (a) Find the constant c;
  (b) compute its distribution function F(t) and draw its graphic;
  (c) compute the mean of T;
  (d) compute the probability that a flashlight lasts for more than 500 hours.

$\big($Answer: a) $c = 2(100)^2$; b) $F(t) = 1 - (100/t)^2$ if $t > 100$ and null otherwise; c) $E(T) = 200$; d) $(0.2)^2$ $\big)$