

BEMACS Bocconi
30401 MATHEMATICS AND STATISTICS - MODULE 2 (STATISTICS)
A.A. 2019/20

EXERCISES ON CENTRAL LIMIT THEOREM and CHI SQUARE, t and F DISTRIBUTIONS.

- (1) If 10 dices are rolled, approximate the probability that the sum of the values obtained (which ranges between 10 and 60) is
- a) between 30 and 40 inclusive;
 - b) less than 30;
 - c) between 30 and 40, given it is greater or equal to 30.
- (Answer: a) 0.6922; b) 0.1539; c) 0.8181)
- (2) The number of people that enter in a drugstore in an hour is a Poisson random variable with parameter $\lambda = 10$. Suppose that the number of clients per hour has the same distribution for any of the hours the drugstore is open (from 8am to 8pm). Assuming that the number of clients in different hours is independent, compute:
- a) the probability that in a given hour at most 3 people enter;
 - b) the probability that the first person enters after the first half an hour (thus in the first half hour nobody enters);
 - c) the probability that today 5 people entered from 8am to 9am, given that 15 people have entered before 10am;
 - d) the probability that in each of 3 consecutive hours more than 3 people enter;
 - e) the approximate probability that in a day the number of clients of the drugstore is between 100 and 140 (inclusive).
- (Answer: a) 0.0103; b) 0.0067; c) $\binom{15}{5} \left(\frac{1}{2}\right)^{15}$; d) 0.9694; e) 0.9386)
- (3) Approximate the probability that the sum of 16 i.i.d. random variables uniform on the interval (0,1)
- a) exceeds 10;
 - b) is between 6 and 10.
- (Answer: a) 0.0418; b) 0.9164)
- (4) When shooting at a target in a 2-dimensional plane, suppose that the horizontal miss distance is normally distributed with mean zero and variance 4 and independent of the vertical miss distance which is also normally distributed with mean zero and variance 4. Let D denote the distance between the point at which the shot lands and the target. Find the expected value of D . Compute:
- a) using the table, the approximate value of the probability that D is greater than 6;
 - b) compare with the exact value of the probability that D is greater than 6;
 - c) the expected value of D .
- (Answer: a) 0.01; b) 0.0111; c) $2\sqrt{2\pi}$)
- (5) If X and Y are two independent Chi square random variables with 3 and 6 degrees of freedom, respectively, determine with the use of the table the probability that $X + Y$ will exceed 19.
- (Answer: $\mathbb{P}(X + Y > 19) \cong 0.025$)

- (6) According to the Census Bureau data, the 9.96% of the American adults (18 years old or older) are Hispanic. In a social survey a random sample of 1200 adults has been interviewed.

- a) What is the average number of Hispanic people interviewed?
- b) What is the probability that in the random sample there are less than 100 Hispanic people? (Use the normal approximation).

(Answer: a) 119.52; b) 0.0268 almost)

- (7) In a telephone survey it is known that the 20% of people contacted finish the questionnaire. Imagine in a survey the caller contacted a sample of 150 people. Let X be the number of the ones who completed the survey.

- a) What is the distribution of X ? What is the expected number of filled questionnaires? and which is the standard deviation?
- b) Approximate the probability that at least 20 questionnaires had been filled.

In a particular survey on bank accounts, the probability that a questionnaire is completed is only $p = 0.08$.

- c) Approximate the probability that at least 20 questionnaires had been filled in this case.
- d) Compute the probability that the number of filled questionnaires is less than 5 using the Poisson approximation. Compare the result with the one obtained with the normal approximation.

(Answer: a) $X \sim \text{binom}(150, 0.2)$; $\mu = 30$; $\sigma = 4.899$; b) $\mathbb{P}(X \geq 20) = 0.9838$ with $p = 0.2$; c) $\mathbb{P}(X \geq 20) = 0.0119$ with $p = 0.08$; d) $\mathbb{P}(X < 5) \cong 0.0076$ (Poisson); $\mathbb{P}(X < 5) \cong 0.0119$ (normal))

- (8) The salaries of pediatric physicians are approximately normally distributed. If 25% of these physicians earn below 180 (thousands US\$) and 25% earn above 320, what fraction earn

- a) below 250?
- b) between 260 and 300?
- c) between 200 and 300?

Consider 10 of these physicians, what are the probability that

- d) 3 of them have a salary below 320?
- e) their average salary is between 200 and 300?

(Answer: a) 0.5; b) 0.1446; c) 0.3688; d) 0.003; e) 0.8714)

- (9) The weekly demand for a product has approximately a normal with mean 1000 and standard deviation 200. The current on hand inventory is 2200 and no deliveries will be occurring in the next two weeks. Assuming that the demands in different weeks are independent,

- a) what is the probability that the demand in each of the next 2 weeks is less than 1100?
- b) what is the probability that the total of the demands in the next two weeks exceeds 2200?
- c) what is the probability that in a year there are more than 40 weeks in which the demand is less than 1100?

(Answer: a) 0.4782; b) 0.2389; c) 0.0869)

- (10) The probability of error in the transmission of a binary digit over a communication channel is $\frac{1}{10^3}$. Denote by N the number of errors when transmitting a block of 10^3 bits. Assume independence.

- a) Write an expression for the exact probability of N greater than 3 (more than 3 errors);
- b) What is the approximate value?

(Answer: a) $1 - \sum_{k=0}^3 \binom{1000}{k} (0.001)^k (0.999)^{1000-k}$; b) 0.0062)

- (11) Suppose the 45% of the population favors a certain candidate in an upcoming election. If a random sample of size 200 is chosen, find

- a) the expected value and standard deviation of the number of members of the sample that favor the candidate;
- b) the probability that more than half the members of the sample favor the candidate.

(Answer: a) $\mu = 90$ and $\sigma = 7.0356$; b) 0.0681)

- (12) If T has a t -distribution with 8-degrees of freedom, using the table, find

- a) $\mathbb{P}(T \geq 1.397)$;
- b) $\mathbb{P}(T \leq 2.306)$;
- c) $\mathbb{P}(-2.306 \leq T \leq 2.306)$

(Answer: a) 0.1; b) 0.975; c) 0.95)

- (13) If X is a Chi square random variables with 6-degrees of freedom, find

- a) $\mathbb{P}(X \leq 6)$;
- b) $\mathbb{P}(3 \leq X \leq 9)$;
- c) $\mathbb{P}(X \leq 6 | 3 \leq X \leq 9)$

(Answer: a) 0.5768 ; b) 0.635; c) 0.61)

- (14) Show that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

(Hint: Recall $\Gamma(\frac{1}{2}) = \int_0^\infty x^{-\frac{1}{2}} e^{-x} dx$ and use the change of variable $x = \frac{z^2}{2}$.)

- (15) If T_n has a t -distribution with n degrees of freedom, show that T_n^2 has an F - distribution with 1 and n degrees of freedom.

- (16) The time it takes a central processing unit to process a certain type of job is normally distributed with mean 20 seconds and standard deviation 3 seconds. If a sample of 15 such jobs is observed, compute the probability that

- a) the sample mean of the times observed is below 19 seconds;
- b) the sample variance will exceed 15.

(Answer: a) 0.0985; b) (approximately) 0.05)