

Exercise 13.1: Population Codes

Consider the following neural network with population codes (see fig. 1(a)): Imagine your mobile phone ringing somewhere close to you where you can see it. If you want to grab it, you need to have an internal representation of where it is relative to you. Say, all you need to know to successfully grab it is the relative position to your head. You can't measure that directly, but you have some cues: you see the mobile at a certain angle (r), you know in which direction you are looking (e) and you know from which direction the sound comes from (a).

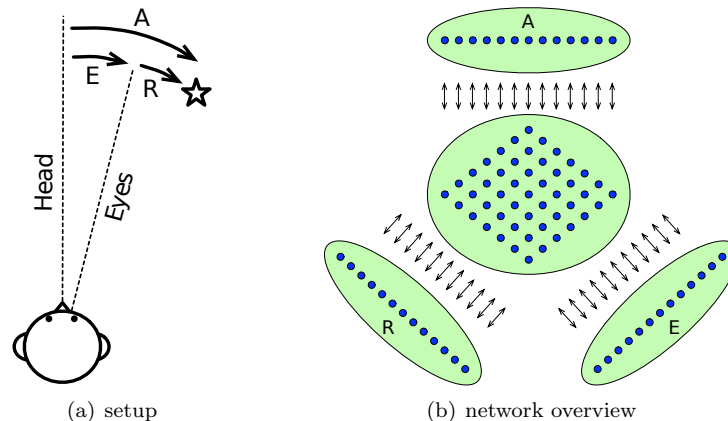


Figure 1: Example scenario for population codes. Concept from Deneve S, Latham PE, Pouget A (2001). Efficient computation and cue integration with noisy population codes. *Nat Neurosci*; 4(8): 826-31.

Let's assume that these three quantities r , e , and a are encoded in three populations R , E , and A (see fig. 1(b)): r is the angle of the mobile on the retina, *i.e.*, the eye-centered position of the mobile; e is the eye position (which angle); and a is the position of the mobile relative to the head as you can measure it from auditory input (head-centered position). For simplicity, we are only considering horizontal angles here.

1. Analytically, there is a relation between the three quantities r , e , and a . What is it?
2. Assume the eye position e is -20° and the image on the retina r is at 10° . The mobile is ringing. Sketch the activity of the populations R , E , and A .
3. Even if the mobile is not ringing, the population A shows the correct position relative to the head. Explain how this can be made possible by setting the weights between the populations and the intermediate layer.
4. Let again the eye position e be -20° and the image on the retina r be at 10° . Using your solution to the previous questions, draw the activity of the intermediate layer. (The intermediate layer is a two-dimensional grid of neurons. Imagine the activities of the populations and think about how they are connected to the layer. Which neurons in the intermediate layer would get the most input? If you got it right, you should recognize three bars in the activity pattern. If you only get two, you are most likely missing the recurrent connections (feedback) from A to the intermediate layer.)
5. The mobile is also vibrating and therefore slowly moving to the left, starting at $a = 40^\circ$ to $a = -40^\circ$. You are too surprised to react in any way – you just keep staring in one direction $e = -20^\circ$. How would you expect the activity of the intermediate layer to change? (Hint: Explain it in terms of the bars in the activity pattern.)
6. Imagine you can record from a single neuron in the intermediate layer as the mobile is moving. Draw the activity of two neurons with respect to a , while $e = -20^\circ$ is fixed. Pick one neuron that has a strong connection to the -20° neuron in E and one that has a weak connection. Explain your observations.
7. Instead of the simple relation between r , e , and a , let's say we want to learn a more complicated one: $a^2 = r^2 + e^2$. Is this still possible? If yes, how would you set the weights? If no, why can't we do it?