Microeconomics
Problem Set 1

Bocconi University Group 30

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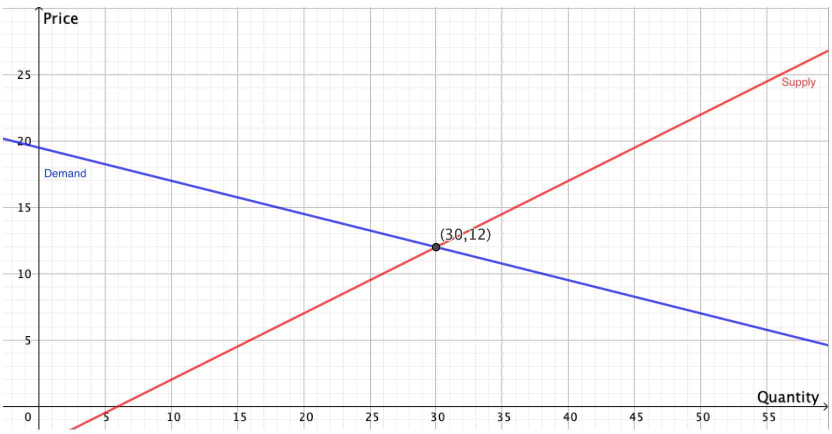
Problem 1

1. At the equilibrium, the quantity supplied is equal to the quantity demanded, therefore we can obtain the equilibrium price and the quantity at the equilibrium by equalling:

$$Q_D = Q_S$$
$$Q_D = 78 - 4p$$
$$Q_S = 6 + 2p$$
$$78 - 4p = 6 + 2p$$
$$p* = 12 \text{ and } Q* = 30$$

2. 

3. The price elasticity of demand is an economic measure of the change in the quantity demanded of a product in relation to a percentage change in the price p. We obtain it's value through the formula:

$$\varepsilon = \frac{\Delta Q/Q}{\Delta p/p} = \frac{\partial Q}{\partial p} * \frac{p}{Q}$$

Introducing the values obtained:

$$\varepsilon = (-4) * \frac{12}{30} = -1.6$$

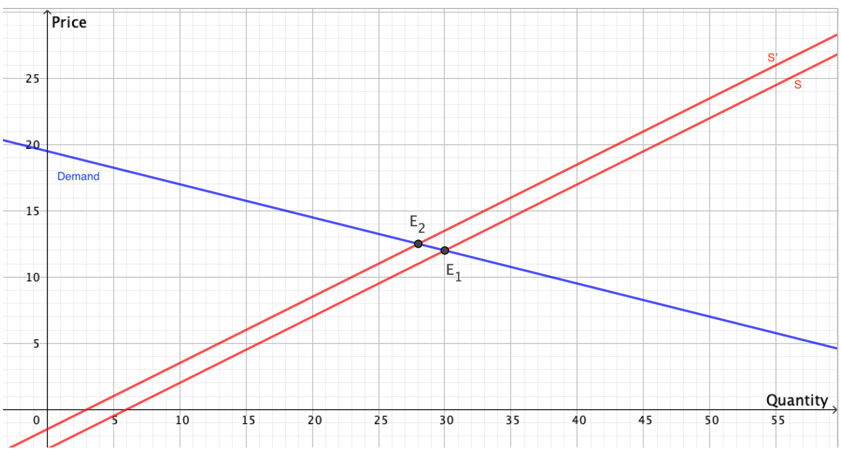
The fact that $|\varepsilon| > 1$ implies that a change of 1 percent in price causes a variation of 1.6 percent in demand. Therefore it can be defined as elastic.

4. The supply curve is shifted to the left due to a drop in production of coffee beans. From the figure below we can evaluate that the supply curve is shifted up and the equilibrium point changes from E_1 to E_2, the price rises and the quantity demanded decrease.

$$S' : Q'_S = 3 + 2p$$

The new equilibrium is:

$$Q'_S = Q_D$$
$$3 + 2p = 78 - 4p$$
$$p* = 12.5$$
$$Q* = 3 + 2 * (12.5) = 28$$



5.

6. The Total Expenditure is the product of price and quantity:
$$TotalExpenditure_1 = 12 * 30 = 360$$

After the unexpected heat wave the:

$$TotalExpenditure_2 = 12.5 * 28 = 350$$

7. The Total Expenditure is decreased and this answer is consistent with the elasticity of demand because the negative (< -1) elasticity of demand states that to an increase of 1 percent causes more than 1 percent decrease in demand. In other words the percent change in price is more than offset by the percent change in the quantity demanded.

8. The government imposed a price ceiling at 12.25. At a price of 12.25:

$$Q_D = 78 - 4(12.25) = 29$$
$$Q'_S = 3 + 2(12.25) = 27.5$$

The quantity demanded is now more than the quantity that firms are willing to supply.

9. This policy intervention will result in a shortage, because there will be an excess of demand over the quantity supplied. Quantitatively speaking there will be a gap between demand and supply of

$$\Delta Q = Q_D - Q'_S = 29 - 27.5 = 1.5 \text{ lb of coffee}$$

Problem 2

1. At the equilibrium, the quantity supplied is equal to the quantity demanded, therefore we can obtain the equilibrium price and the quantity at the equilibrium by equalling:

$$Q_D = Q_S$$
$$Q_D = 1000 - 50p$$
$$Q_S = 840 + 110p$$
$$1000 - 50p = 840 + 110p$$
$$p* = 1 \text{ and } Q^* = 950$$

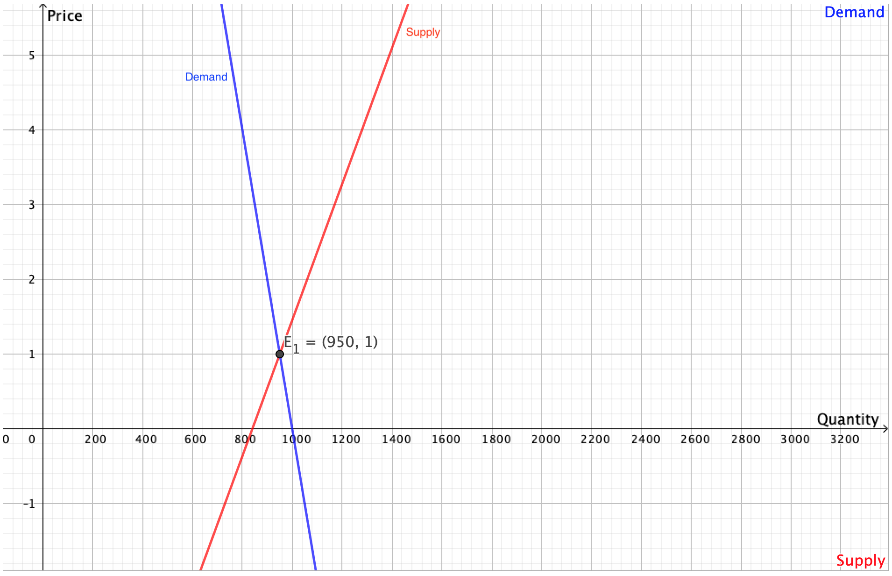
2. The government of the U.S. it's imposing a tax on sugar - sweetened drinks. The taxes can be imposed both on consumers and producers, causing the shift of demand functions or supply functions. Let's suppose the government collects the tax from suppliers (as we will see later, the equilibrium will be the same regardless of whether the government collects the tax from customers or suppliers).

$$S' : Q'_S = 110(p - 0.5) + 840 = 785 + 110p$$

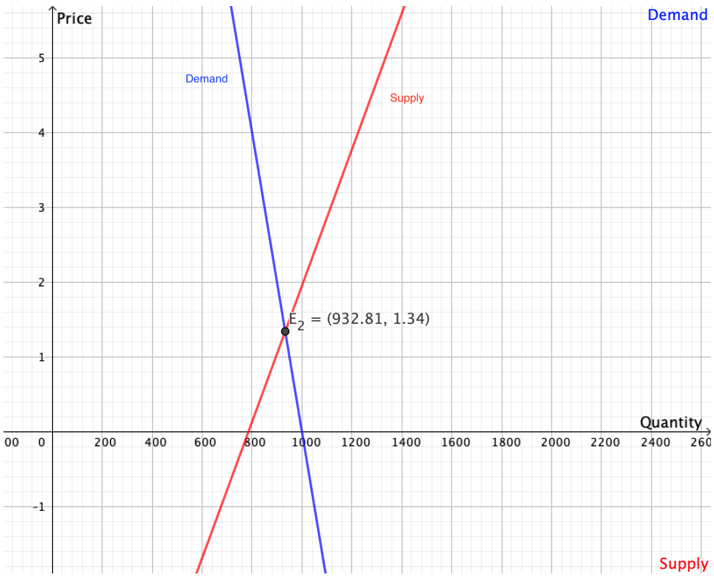
Let's now calculate the new equilibrium conditions, where p_C is the price paid by consumers:

$$Q'_S = Q_D$$
$$785 + 110p = 1000 - 50p$$
$$p_C = 1.34 \simeq 1.3$$
$$Q^* = 932.81 \simeq 933$$

3. The graph that follow shows the market equilibrium without the imposition of the tax, where E_1 is the point of equilibrium:



The graph that follow shows the market equilibrium with the imposition of the tax on suppliers, where E_2 is the point of equilibrium:



4. From the equilibrium situation we can evaluate that consumers are willing to pay **1.3** for a quantity of 933; while due to the taxation, the producers receive an amount equal to:

$$1.3 - 0.5 = 0.8$$

5. To show that there is no difference between applying the tax on consumers or suppliers we can suppose that the tax is imposed on buyers and see if the results are the same. If the taxation is applied on consumers, they'll have to pay the following price:

$$p_t = (p + t) = 1 + 0.5$$
$$D' : Q'_D = 100 - 50(p + 0.5) = 975 - 50p$$

We can now evaluate the new equilibrium, where p_S is the price received by firms:

$$Q_S = Q'_D$$
$$840 + 110p = 975 - 50p$$
$$p_S = 0.84 \simeq 0.8$$
$$Q^* = 932.81 \simeq 933$$

The market equilibrium has the same conditions as the ones evaluated with taxation applied on producers, therefore there is no difference if the tax is imposed on buyers or on sellers.

6. The incidence of the "fat-tax" on consumers is (since Number 2 asked for rounded prices, we also used in the following the rounded prices) :

$$\frac{|p-p_C|}{tax} = \frac{|1-1.3|}{0.5} = 60\%$$

The incidence of the "fat-tax" on producers is:

$$\frac{|p-p_S|}{tax} = \frac{|1-0.8|}{0.5} = 40\%$$

7. The results obtained in evaluating the inferences on consumers and producers inform us that a larger proportion of the tax falls on buyers. This tells us that the supply function is more elastic than the demand function of buyers.
We can double check our theory by evaluating the demand elasticity:

$$\frac{\delta Q/Q}{\delta p_C/p} \; \frac{(933-950)/950}{(1{,}34-1)/1} \; \frac{-17/950}{0{,}34} = -0,05$$

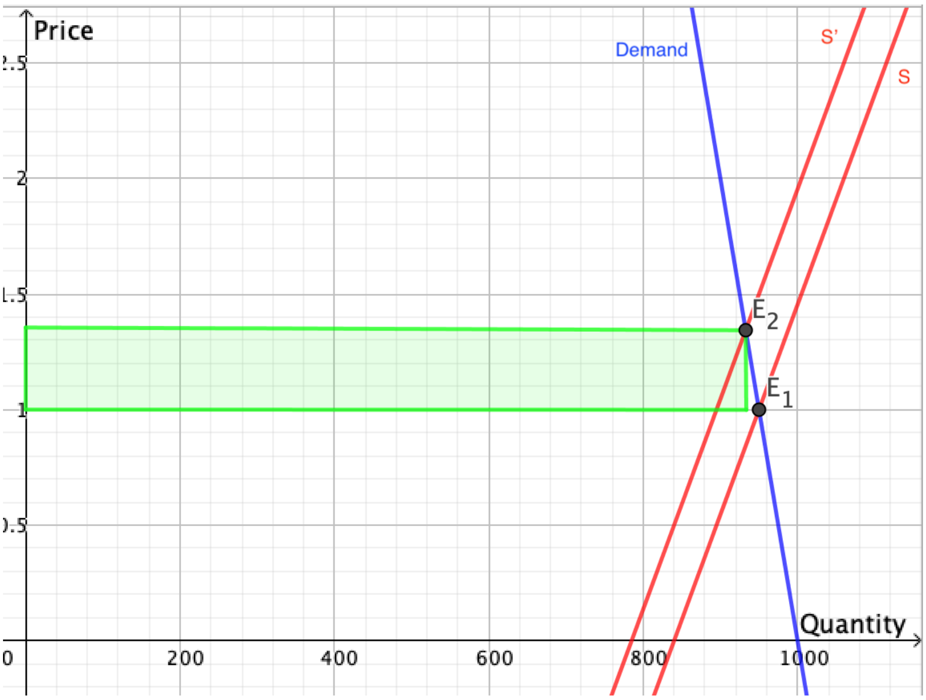
And the supply elasticity:

$$\frac{\delta Q/Q}{\delta p_S/p} \; \frac{(933-950)/950}{(0{,}84-1)/1} \; \frac{-17/950}{-0{,}16} = 0,11$$

8. The government earns an amount equal to:

$$0.5 * 933 = 466.5$$

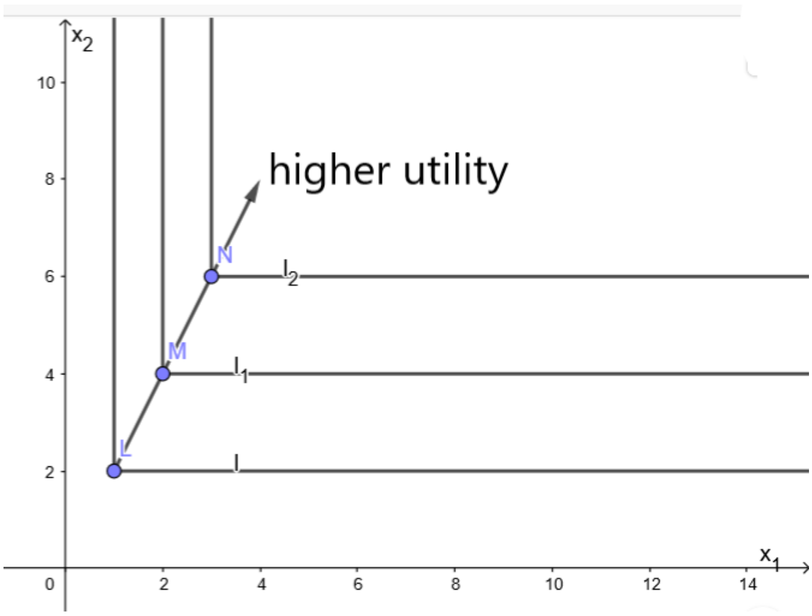
The graphical representation of the amount earned by the government through the imposition of this tax is the following polygon:



Problem 3

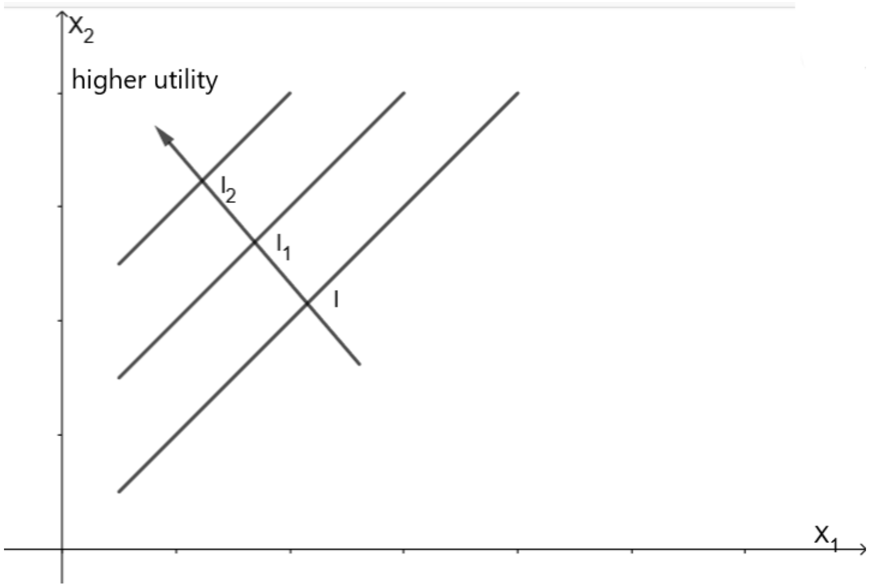
1. When he reads a book x_1 in the evening, Sean Connery always sips two cups of Rooibos tea, x_2.

$$x_1 = \text{book} \qquad x_2 = 1 \text{ cup of tea} \rightarrow \text{Perfect complements}$$
$$U(x_1, x_2) = min(2x_1, x_2)$$



2. Roger Federer does not like Ronzoni pasta, x_1, but loves Barilla pasta, x_2.

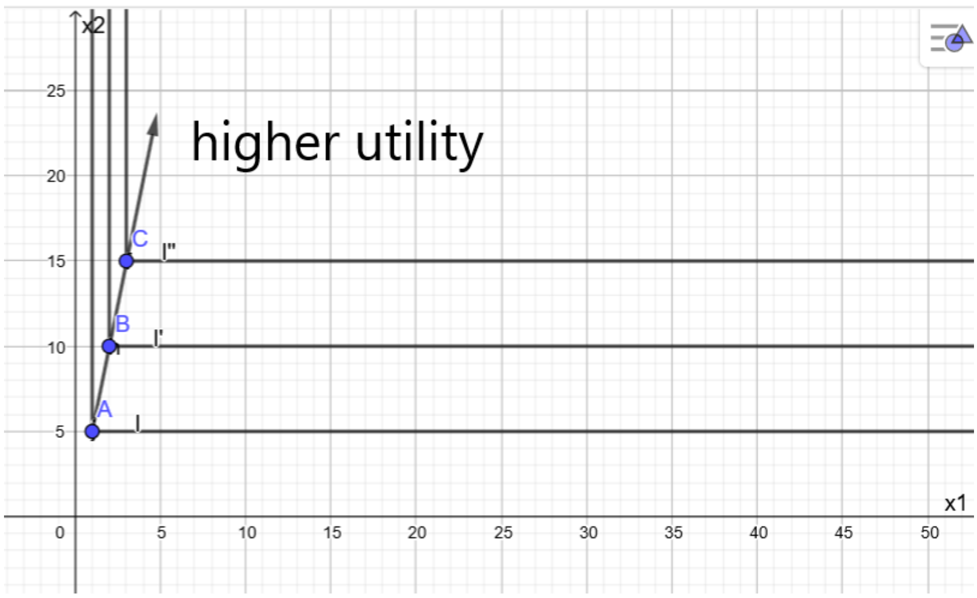
$$x_1 = \text{Ronzoni pasta} \qquad x_2 = \text{Barilla pasta}$$
$$\text{Federer strictly prefers } x_2 \text{ to } x_1 \rightarrow x_2 > x_1$$
$$U(x_1, x_2) = x_2 - x_1$$



3. When Tina Turner eats 1 slice of apple pie, x_1, she always adds (and eats) 5 scoops of icecream, x_2, on the top of the slice of the pie (golosona!!!).

x_1= 1 slice of apple pie x_2= 1 scoop of icecream $\rightarrow$ Perfect complements

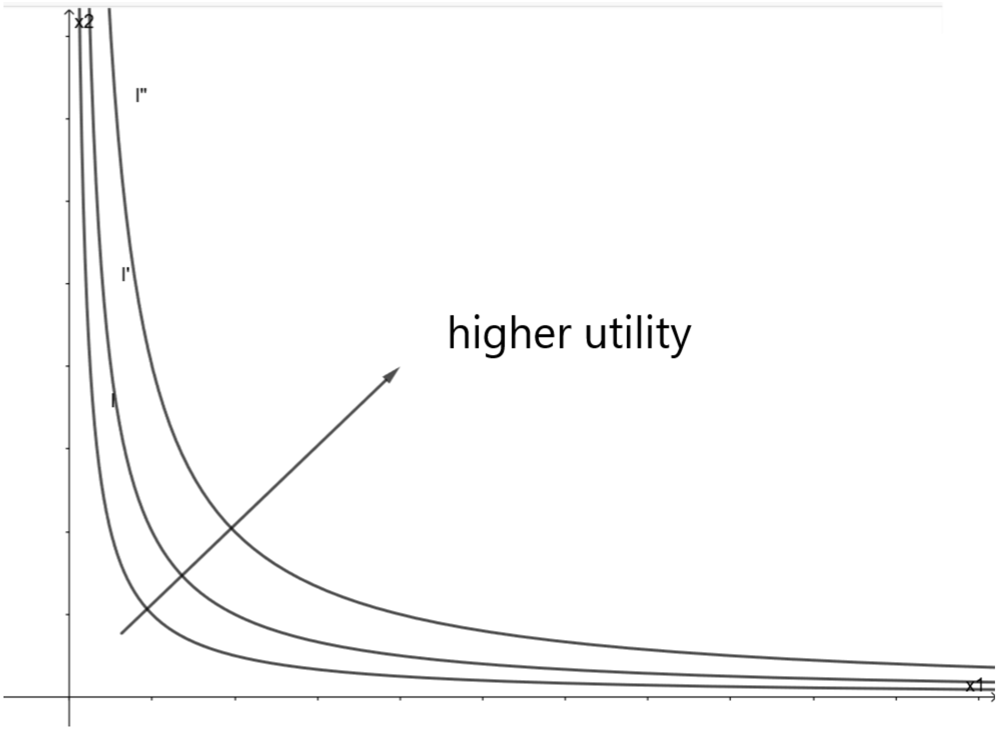
$$U(x_1, x_2) = min(5x_1, x_2)$$



4. For breakfast, J-Ax has toasted bread, x_1, and jam, x_2. Consuming more of one good makes consuming the other good more enjoyable.

x_1= toasted bread $\qquad$ x_2= jam

$$U(x_1, x_2) = x_1 x_2$$

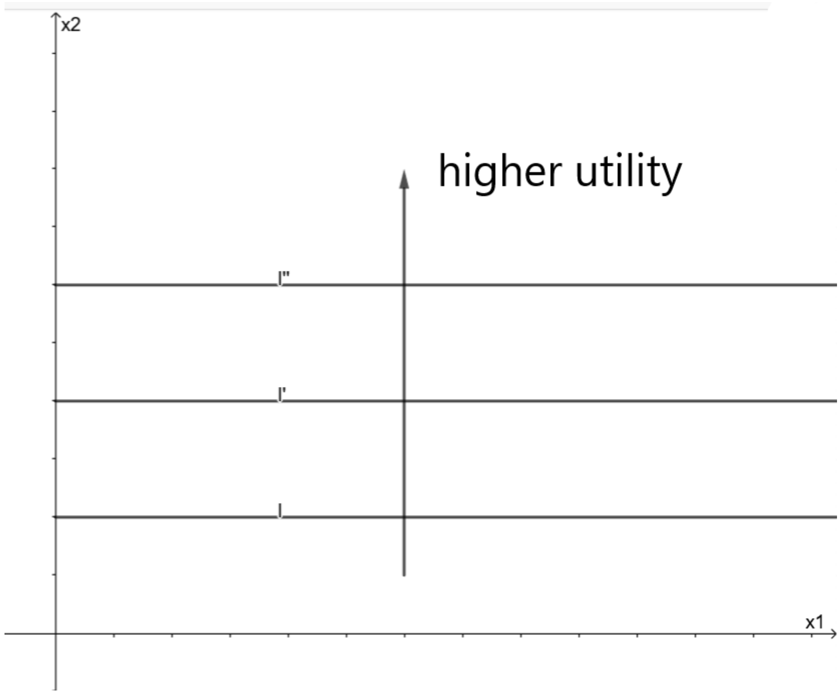


5. Scarlett Johansson neither likes nor dislikes pasta all'amatriciana,

x_1, but she loves desserts, x_2.

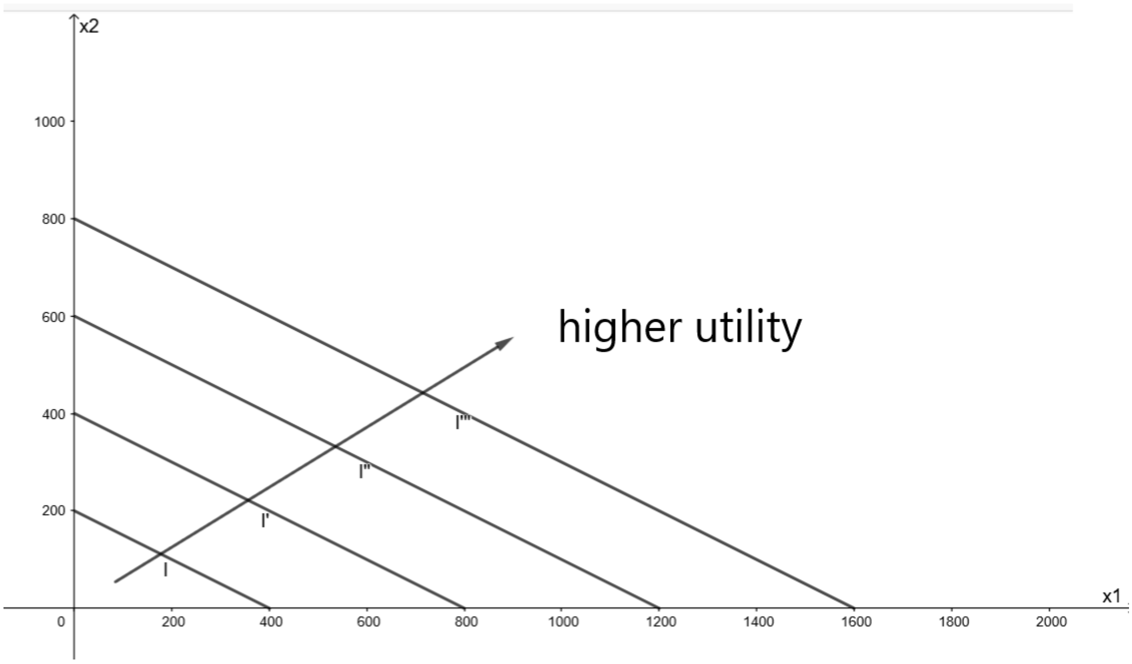
x_1= pasta all'amatriciana $\qquad$ x_2= desserts

$$U(x_1, x_2) = x_2$$



6. It requires 400 milligrams of medicine x_1 to obtain the same pain relief one gets with 200 milligrams of medicine x_2.

$$x_1\text{= medicine(1)} \qquad x_2\text{= medicine(2)} \ \rightarrow \ \text{Perfect substitutes}$$
$$U(x_1, x_2) = 200x_1 + 400x_2 = x_1 + 2x_2$$



Problem 4

Filippo has just started an internship. He decides to buy new elegant shirts, x_1, and professional ties, x_2. The price of an elegant shirt is $p_1 = \$160$ and the price of a tie is $p_2 = \$80$. Suppose Filippo's total budget for renewing his wardrobe, thus buying shirts and ties, is $I = \$640$. His utility function can be written as follows:

$$U(x_1, x_2) = \sqrt{x_1 x_2}$$

1. Define the marginal rate of substitution, MRS.
   The **marginal rate of substitution (MRS)** is the rate at which a consumer can give up some amount of one good in exchange for another good while maintaining the same level of utility. It is defined as the **slope** at a particular point on an indifference curve.

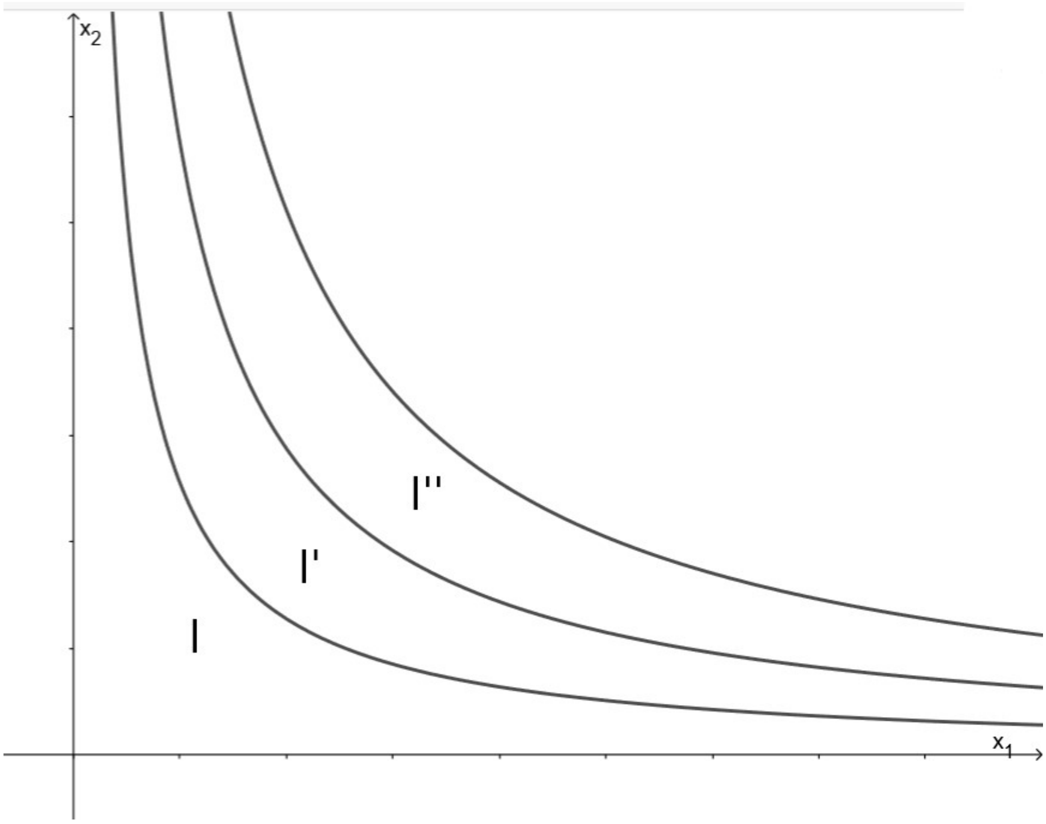
2. Calculate Filippo's MRS.

$$MRS(x_1, x_2) = -\frac{MUx_1}{MUx_2}$$

$$U_1 = -\tfrac{\partial U}{\partial x_1} = -(\tfrac{1}{2}(x_2x_1)^{-\frac{1}{2}} \cdot \tfrac{d}{dx_1}(x_2x_1)) = -\tfrac{x_2}{2\sqrt{x_1x_2}}$$
$$U_1 = -\tfrac{\partial U}{\partial x_2} = -\tfrac{x_1}{2\sqrt{x_1x_2}}$$

$$MRS(x_1, x_2) = -\frac{-\frac{x_2}{2\sqrt{x_1x_2}}}{-\frac{x_1}{2\sqrt{x_1x_2}}} = -\frac{x_2}{x_1}$$

3. On a labeled graph, draw at least two indifference curves corresponding to Filippo's utility function.



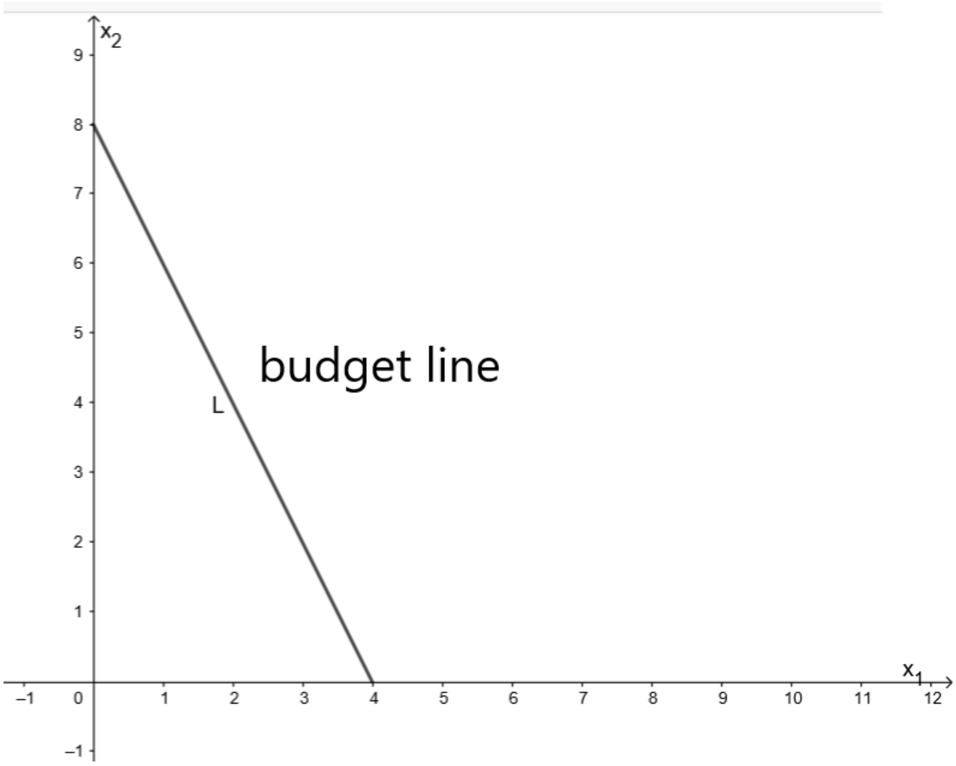
4. Define what the budget constraint is.
   BUDGET CONSTRAINT:the bundles of goods that can be bought if a consumer's entire budget is spent on those goods at given prices.
   Consumers maximize their well-being subject to constraints; the budget constraint is the most important constraint that most of consumers face in deciding what to consume.

$$p_1q_1 + p_2q_2 = Y \rightarrow \textbf{Budget line} \text{ or } \textbf{Budget constraint}$$

5. Write Filippo's budget constraint.

$$160x_1 + 80x_2 = 640$$

6. Show Filippo's budget constraint on the graph.



7. Find analytically the optimal consumption bundle $x^* = (x_1^*, x_2^*)$ for Filippo if he decides to buy now shirts and ties. Is it an interior or corner solution? Explain why.

$$\begin{cases} \mid MRS \mid = \frac{p_1}{p_2} = \frac{x_2}{x_1} \\ BC : p_1 x_1 + p_2 x_2 = Y \end{cases}$$

$$\begin{cases} \frac{x_2}{x_1} = 2 \\ 160x_1 + 80x_2 = 640 \end{cases}$$

$$\begin{cases} x_2 = 2x_1 \\ 2x_1 + x_2 = 8 \end{cases}$$

$$x_1^* = 2;$$

$$x_2^* = 4$$

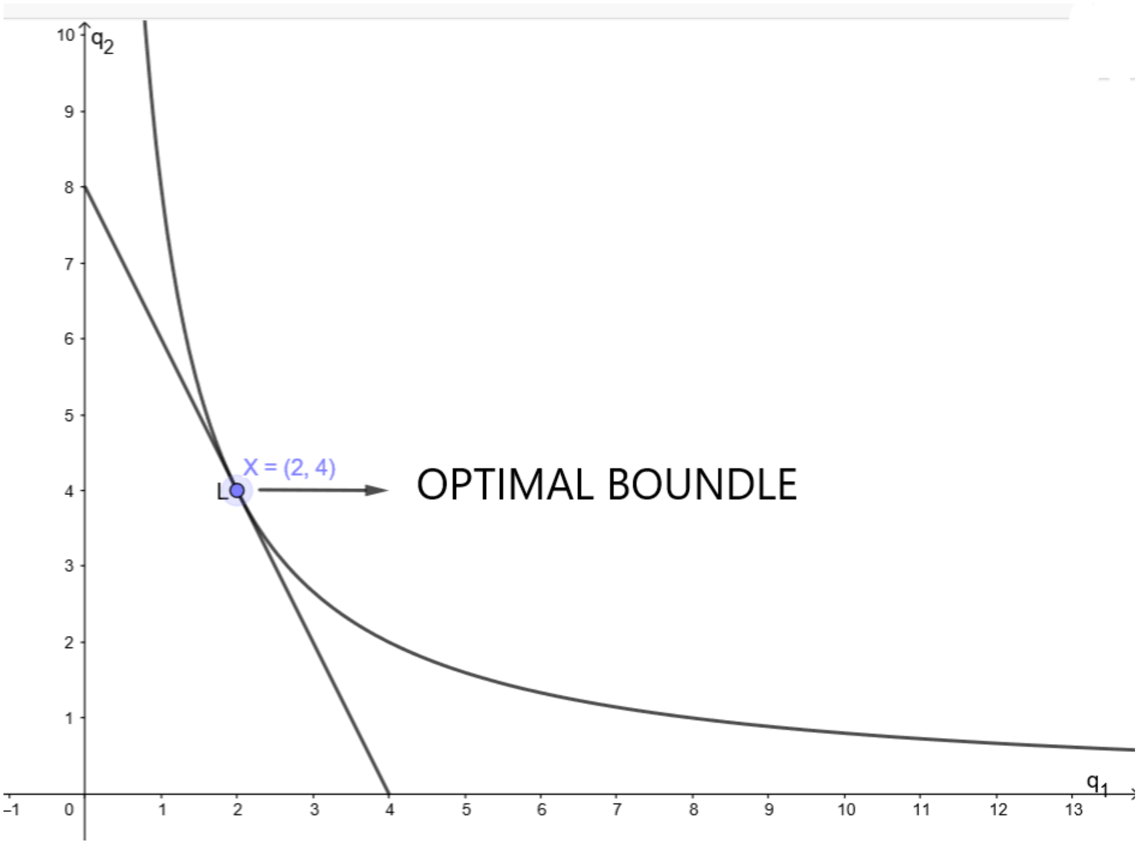
$$x^*(x_1^*, x_2^*) = (2, 4)$$

This is an interior solution because Filippo prefers to consume a positive quantity of both goods in his optimal bundle. The utility function is a Cobb-Douglas utility function, therefore it always have interior solution since, if one good was 0, then the function would be equal 0.

8. Calculate Filippo's utility level when he choses x^*

$$U = \sqrt{x_1 x_2} = \sqrt{2 \cdot 4} = \sqrt{8} = 2\sqrt{2}$$

9. Show Filippo's optimal choice on the graph.



10. Suppose that Filippo actually decided to wait buying shirts and ties. When he finally decides to buy new shirts and ties, he gets very disappointed: the price for an elegant tie is now 75% more than its original price!!! Find analytically Filippo's new optimal choice, $x^{**} = (x_1^{**}, x_2^{**})$, rounding the quantities to the nearest integer.

We find the new optimal bound $x^{**}(x_1^{**}, x_2^{**})$ using the privious method and replacing p_1 with p_1^*.

$$\begin{cases} \mid MRS \mid = \frac{p_1^*}{p_2} = \frac{x_2}{x_1} \\ BC : p_1^* x_1 + p_2 x_2 = Y \end{cases}$$

$$\begin{cases} \frac{x_2}{x_1} = \frac{160}{140} \\ 160x_1 + 140x_2 = 640 \end{cases}$$

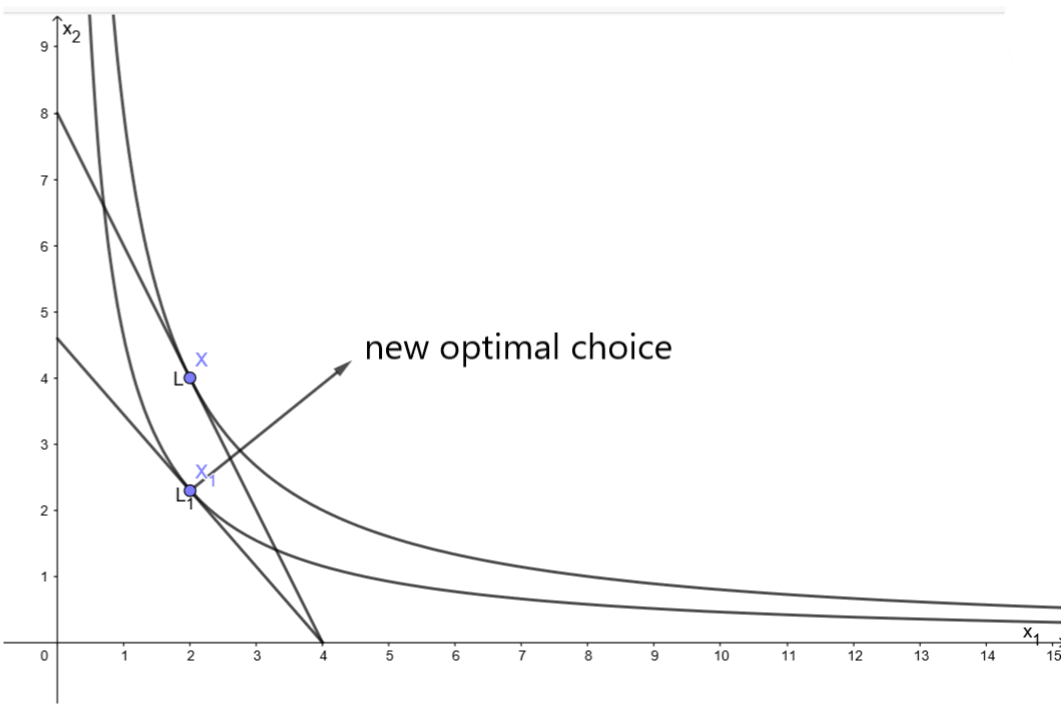
$$\begin{cases} x_2 = \frac{8}{7}x_1 \\ 8x_1 + 7x_2 = 32 \end{cases}$$

$$x_1^* = 2$$

$$x_2^* = 2.29 = 2$$

$$x^{**}(x_1^{**}, x_2^{**}) = (2, 2)$$

11. Use the same graph to show how Filippo's optimal choice is modified.



12. Is Filippo changing the optimal bundle? What happens to the consumption of shirts, x_1? Why?

Filippo **is** changing the **optimal bundle**.

$$x^*(x_1^*, x_2^*) = (2,4) \rightarrow \quad x^{**}(x_1^{**}, x_2^{**}) = (2,2)$$

He is changing the optimal bundle because he could not afford the first bundle anymore due to the increasing of ties' price. In a Cobb-Douglas function the good x_1 will be unchanged because the demand of x_1 depends only on its own price and the income. Therefore the consumption of shirt remains unchanged since it is not affected by the change of price p_2.

13. What happens to Filippo's utility level? Explain why.

$$U^* = \sqrt{x_1 x_2} = \sqrt{4} = 2$$

The utility level decreases since Filippo gets the same quantity of good x_1 as he did before but he gets less of good x_2.

14. Suppose that on his birthday, Filippo receives a gift card to buy ties and shirts. The value of the gift card is such that Filippo, by using it, reaches a satisfaction (utility) level equal to the one he would have reached before the increase in the price of ties (point 8). Define and find analytically Filippo's expenditure function.

$$\begin{cases} \mid MRS \mid = \frac{p_1}{p_2^*} = \frac{x_2}{x_1} \\ U = \sqrt{x_1 x_2} \end{cases}$$

$$\begin{cases} \frac{160}{140} = \frac{x_2}{x_1} \\ 2\sqrt{2} = \sqrt{x_1 x_2} \end{cases}$$

$$\begin{cases} x_2 = \frac{8}{7} x_1 \\ 8 = x_1 x_2 \end{cases}$$

$$x_1^{***} = 2.6$$

$$x_2^{***} = 3.0$$

Expenditure function: the relationship showing the minimal expenditure necessary to achieve a specific utilty level for a given set of prices.

$$E = p_1^* x_1^{***} + p_2 x_2^{***} = 160 \cdot 2.6 + 80 \cdot 3.0 = 836$$

15. What is the optimal bundle $x^{***} = (x_1^{***}, x_2^{***})$ chosen by Filippo now?

$$x^{***} = x^{***} = (x_1^{***}, x_2^{***}) = (2.6, 3.0)$$

16. Which is the value of the gift card he is provided with? [Hint: round numbers to one decimal place. Don't worry if the numbers of shirts and ties are not integers here!]

$$V_{gc} = E - Y = 836 - 640 = 196$$

Problem 5

1. The MRS can be calculated with the following formula:

$$MRS = -\frac{MU_{x1}}{MU_{x2}} = -\frac{5x_1^{-0.5}}{5} = -x_1^{-0.5}$$

2. The MRS shows the maximum quantity of good x_2 an individual is willing to give up in order to get one additional unit of good x_1 and remain equally happy. So, Harry is willing to give up $x_1^{-0.5}$ units of "Chocolate frogs", x_2, in order to get one additional unit of "Bertie Bott's Every Flavor Beans", x_1.

3. In order to understand how the MRS varies, when good x_1 increases, we have to differentiate the expression with respect to x_1:

$$\frac{\partial|MRS|}{\partial x_1} = -0.5x^{-1.5} < 0$$

   Therefore, the MRS decreases when good x_1 increases. The reasoning behind the movement is the following: Harry loves "Bertie Bott's Every Flavor Beans",x_1, but he also likes "Chocolate frogs",x_2. So, When he already has a lot of "Bertie Bott's Every Flavor Beans", x_1, he is not willing to give up many units of "Chocolate frogs", x_2, in order to get one more "Bertie Bott's Every Flavor Bean". When he only has a few "Bertie Bott's Every Flavor Beans",x_1, then he would give up much more units of "Chocolate frogs", x_2, in order to get one more "Bertie Bott's Every Flavor Bean".

4. Since we have a quasilinear utility function, there may be an internal solution, but also a corner solution. We first check for an interior solution using the tangency condition and the budget constraint. If we find that these conditions imply that Harry wants to buy positive quantities of both goods, we have found an interior solution. Otherwise we have to determine a corner solution.

$$\begin{cases} MRS = MRT \\ BC : I = p_1x_1 + p_2x_2 \end{cases}$$

$$\begin{cases} -x_1^{-0.5} = -p_1 \\ I = p_1x_1 + x_2 \end{cases}$$

$$\begin{cases} x_1 = \frac{1}{p_1^2} \\ x_2 = I - p_1 x_1 \end{cases}$$

$$\begin{cases} x_1^* = \frac{1}{p_1^2} \\ x_2^* = I - \frac{1}{p_1} \end{cases}$$

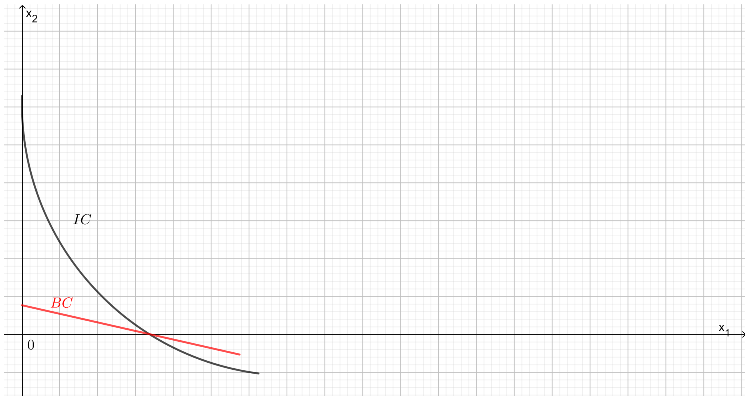
We now see that x_1^* does not depend on Income I. But, x_2^* does depend on I. We also recognize that x_2^* will only be positive when $I > \frac{1}{p_1}$. So, these are the only cases, when we have an interior solution. If $I \le \frac{1}{p_1}$, we have a corner solution. In this cases, how do we know, which good Harry will buy? To answer this question we have to compare the amount of extra utility Harry gets from "Bertie Bott's Every Flavor Beans" per dollar spent on "Bertie Bott's Every Flavor Beans", $\frac{MU_{x_1}}{p_1}$ and the extra utility he gets from "Chocolate frogs" per dollar spent on "Chocolate frogs", $\frac{MU_{x_2}}{p_2}$. If $\frac{MU_{x_1}}{p_1} = \frac{5x_1^{-0.5}}{p_1} > 5 = \frac{MU_{x_2}}{p_2}$ he will only consume good x_1 and vice versa. If $\frac{5x_1^{-0.5}}{p_1} = 5$ he is indifferent between spending all his income on good x_1 or good x_2. Let's summarize:

$$\begin{cases} x_1^* = \frac{1}{p_1^2}, \;\; x_2^* = I - \frac{1}{p_1} & if I > \frac{1}{p_1} \\ x_1^* = \frac{I}{p_1}, \;\; x_2^* = 0 & if \;\; I \le \frac{1}{p_1} \;\; and \;\; \frac{5x_1^{-0.5}}{p_1} \ge 5 \\ x_1^* = 0, \;\; x_2^* = \frac{I}{1} & if \;\; I \le \frac{1}{p_1} \;\; and \;\; \frac{5x_1^{-0.5}}{p_1} \le 5 \end{cases}$$

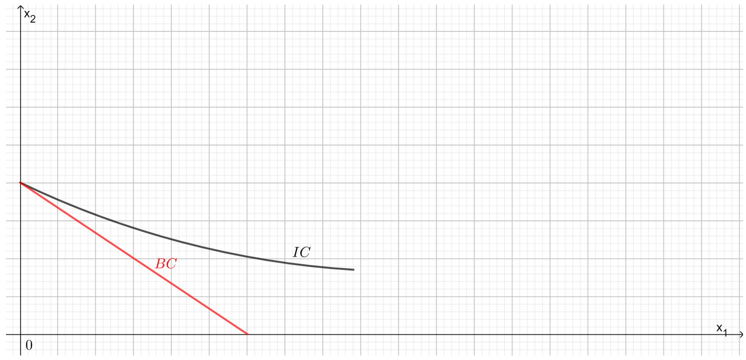
You can't consume 0 X1 because p>0

5. The first figure shows an interior solution:

The second figure shows an corner solution, where only good x_1 is consumed:



The third figure shows an corner solution, where only good x_2 is consumed:



Problem 6

1. Plastic shopping bags x_1 and biodegradable shopping bags x_2 are perfect substitutes. Therefore: $U(x_1, x_2) = ix_1 + jx_2$. In our case, we have to consider that the slope of the indifference curve is not -1. The reason is, that by using one plastic bag, Lucille only gets $\frac{1}{4}$ of the satisfaction she gets from consuming one biodegradable shopping bag. This leads to the following utility function:

$$U(x_1, x_2) = \tfrac{1}{4}x_1 + x_2$$

By calculating the MRS, we can double check, whether this utility function is correct. The MRS shows the maximum amount of good x_2 Lucille is willing to give up in order to get one additional unit of good x_1 and remain equally happy. Since Lucille gets from using plastic bags, x_1 only $\frac{1}{4}$ of the satisfaction she gets from consuming biodegradable shopping bags, x_2, Lucille will only be willing to give up $\frac{1}{4}$ of good x_2 in order to get one additional unit of good x_1. Therefore, the MRS must be $\frac{1}{4}$. So, let's check whether we will get the correct result for the MRS:

$$MRS = -\frac{MU_{x1}}{MU_{x2}}$$
$$MRS = -\frac{1}{4}$$

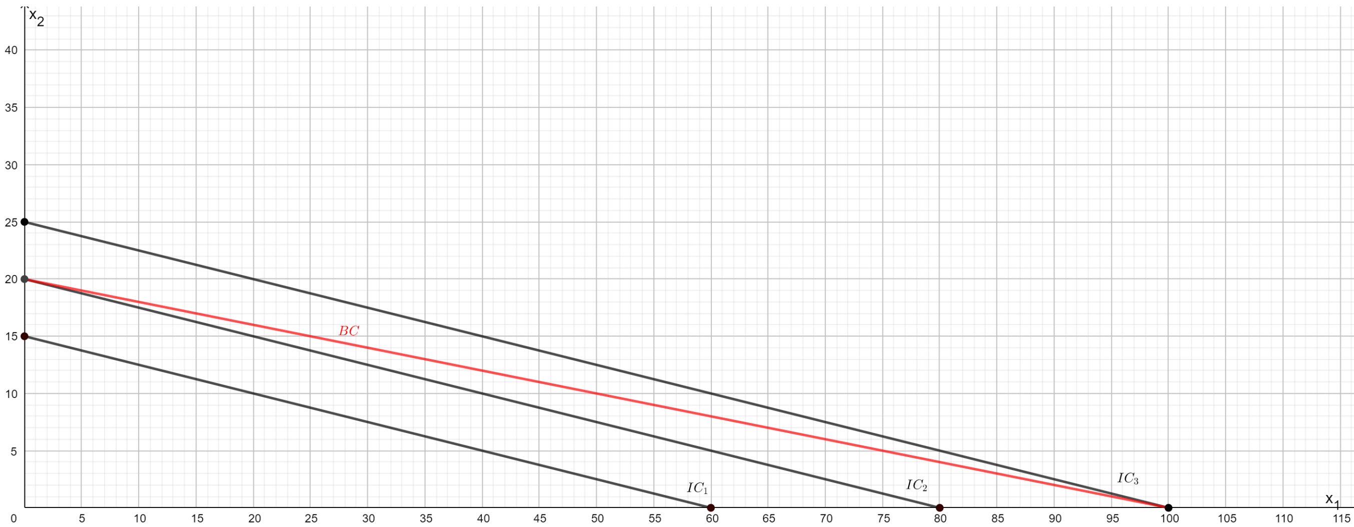
2. As already mentioned above, the MRS is $-\frac{1}{4}$. Therefore the slope of the indifference curves has to be $-\frac{1}{4}$ and we get the following indifference curves:

3. Lucille's budget constraint is:

$$Y = p_1 x_1 + p_2 x_2$$
$$10 = 0.1x_1 + 0.5x_2$$

4. If we rearrange the budget constraint a little bit, we get:

$$x_2 = \frac{Y}{p_2} - \frac{p_1}{p_2}$$
$$x_2 = 20 - 0.2x_1$$



5. First, we have the check, whether there is an interior solution or not. For an interior solution, the MRS must be equal to the MRT.

$$MRS = -\frac{1}{4}$$
$$MRT = -\frac{1}{5}$$
$$\frac{1}{4} \neq \frac{1}{5}$$

Therefore, we know that we have to find a corner solution. In other words, Lucille will maximise her utility, if she only consumes one good. But which one? To answer this question we have to compare the amount of extra utility she gets from plastic bags per dollar spent on plastic bags and the extra utility she gets from biodegradable shopping bags per dollar spent on biodegradable shopping bags. Formally:

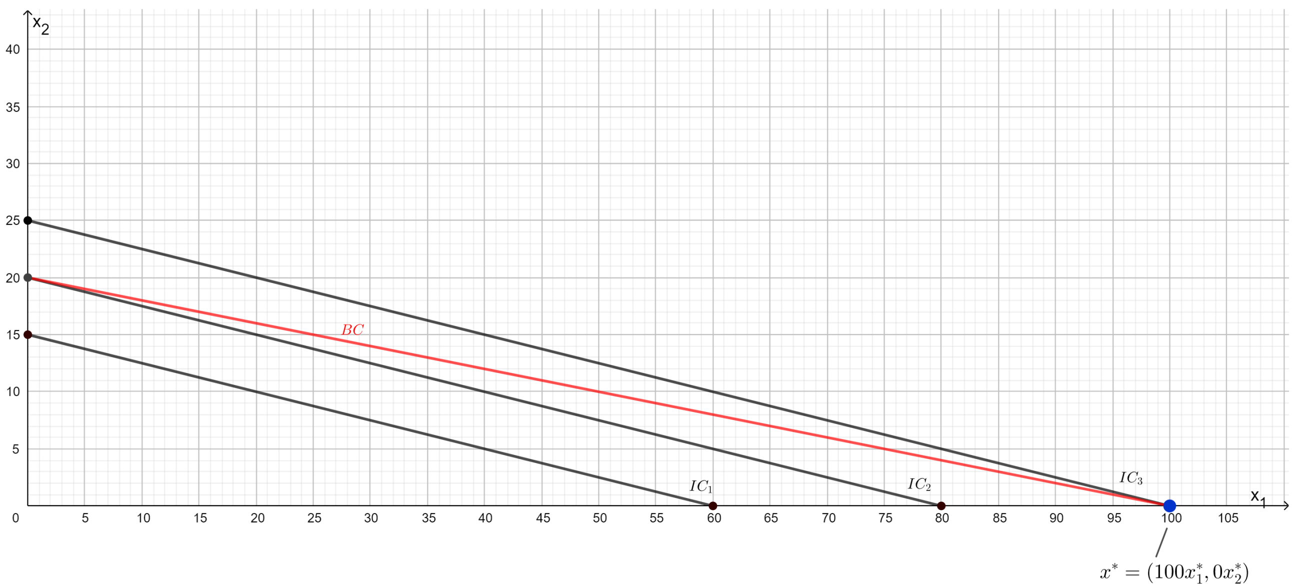
$$\frac{MU_{x1}}{p_1} = \frac{0.25}{0.1} = 2.5$$
$$\frac{MU_{x2}}{p_2} = \frac{1}{0.5} = 2$$
$$\frac{MU_{x1}}{p_1} > \frac{MU_{x2}}{p_2}$$

Therefore, Lucille should spend all her income on plastic bags. With a budget of 10 Dollars, she can afford 100 plastic bags:

$$Y = p_1x_1 + p_2x_2$$
$$10 = 0.1x_1 + 0.5 * 0$$
$$x_1 = 100$$

The optimal bundle is $x^* = (100x_1^*, 0x_2^*)$.

6. This optimal bundle can also be shown in our figure:



7. First, we have to check whether we have an interior solution:

$$MRS = -\frac{1}{4}$$
$$MRT = -\frac{1}{2}$$
$$\frac{1}{4} \neq \frac{1}{2}$$

Again, we will have a corner solution. Lucille should only consume one good. Let's find out which one:

$$\frac{MU_{x1}}{p_1} = \frac{0.25}{0.1} = 2.5$$
$$\frac{MU_{x2}}{p_2} = \frac{1}{0.2} = 5$$
$$\frac{MU_{x1}}{p_1} < \frac{MU_{x2}}{p_2}$$

Now, the extra utility Lucille gets from plastic bags per dollar spent on plastic bags is less than the extra utility she gets from biodegradable shopping bags per dollar spent on biodegradable shopping bags. Therefore she should only buy biodegradable shopping bags. Let's find the new optimal choice:

$$Y = p_1x_1 + p_2x_2$$
$$10 = 0.1 * 0 + 0.2 * x_2$$
$$x_2 = 50$$

Therefore, The optimal bundle is $x^{**} = (0x_1^{**}, 50x_2^{**})$.

8. Now, we have a new budget line. The new optimal bundle is $x^{**} = (0x_1^{**}, 50x_2^{**})$.

Problem 7

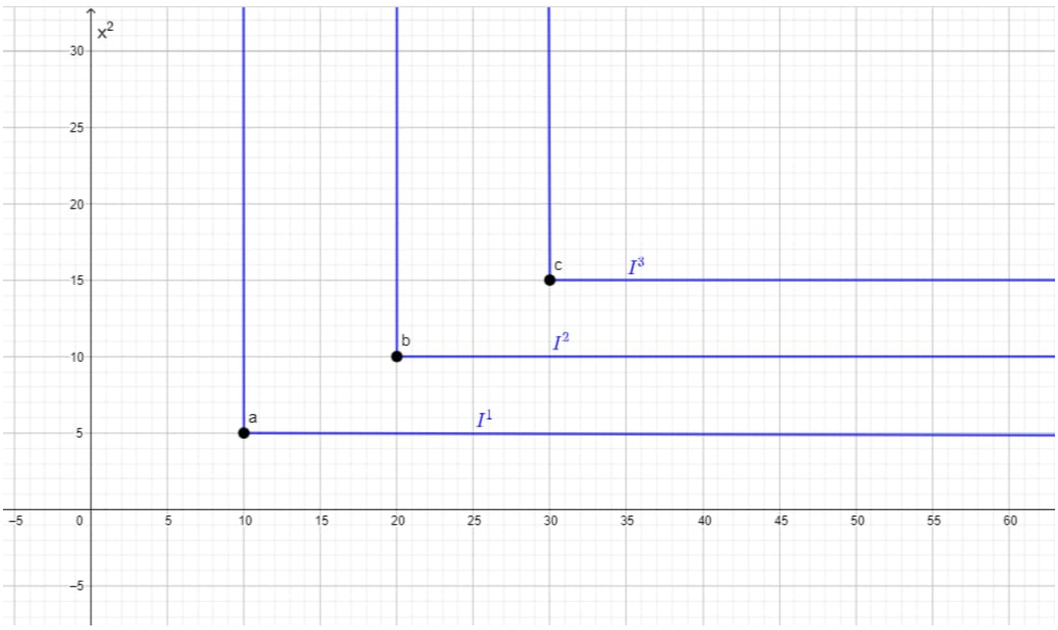
1. In our case the goods are perfect complements, the general form of utility function for perfect complements is:

$$U(x_1, x_2) = min(ix_1, jx_2)$$

The two perfect complements taken in consideration in this problem are headlamp and first-aid kit:

$$x_1 \text{ - headlamp}$$
$$x_2 \text{ - first-aid kit}$$
$$U(x_1,x_2) = min(x_1,2x_2)$$

2. The following picture shows the indifference curves (I_1, I_2, I_3).



3. $Y = p_1x_1 + p_2x_2 \rightarrow 5x_1 + 10x_2 = 200$

4. The budget constraint shows the quantity of x_1 and x_2 Selim can buy at the given prices ($p_1 = 5$ and $p_2 = 10$) and with a fixed income ($Y = 200$).

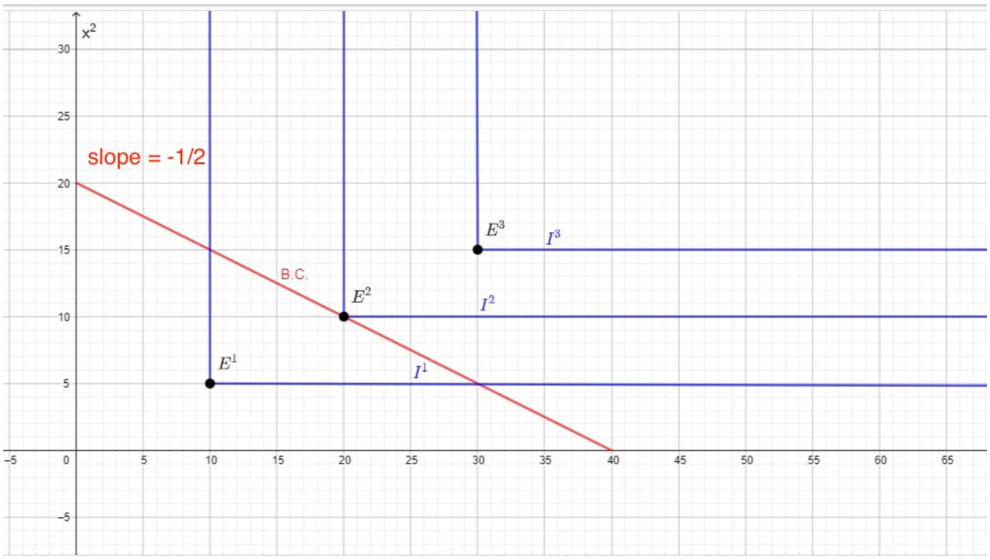
Budget constraint

Horizontal intercept : $x_1 = 40$

Vertical intercept : $x_2 = 20$

Slope : $-\frac{5}{10} = -\frac{1}{2}$

5. The following graph only shows the budget constraint.



6. The slope of the B.C shows how many units of good x_2 must be forgone to buy an additional unit of x_1. This, combined with the finite Y, is why the slope is negative.

7. We know that the optimal bundle lies on the Vertex Line: $x_2 = 0.5x_1$. Therefore, we can calculate the optimal bundle by solving the following maximization problem:

$$\begin{cases} x_2 = 0.5x_1 \\ p_1x_1 + p_2x_2 = I \end{cases}$$

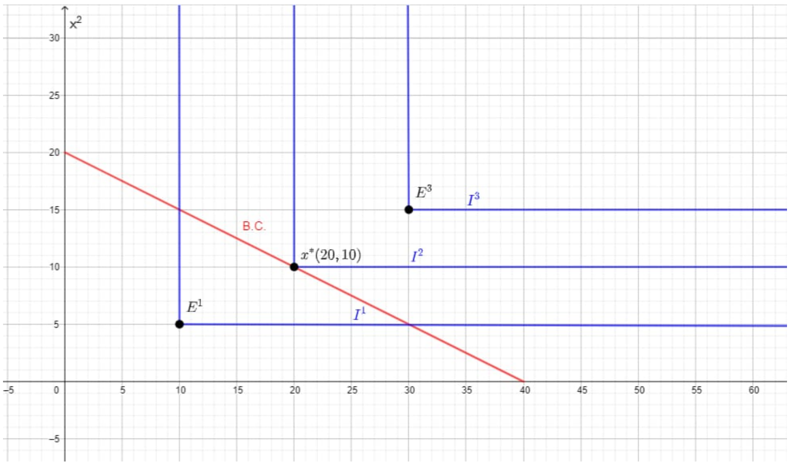
$$\begin{cases} x_2 = 0.5x_1 \\ 5x_1 + 5x_1 = 200 \end{cases}$$

$$\begin{cases} x_1^* = 20 \\ x_2^* = 10 \end{cases}$$

Therefore, the optimal bundle is:

$x^* = (\ x_1^*\ ,\ x_2^*) = (20{,}10)$

8.



Problem 8

1. First, let's calculate the MRS of the two utility functions:

$$MRS_u = -\frac{MU_{x_1}}{MU_{x_2}} = -\frac{0.2x_1^{-0.8}x_2^{0.3}}{0.3x_2^{-0.7}x_1^{0.2}} = -\frac{2x_2}{3x_1}$$
$$MRS_v = -\frac{MU_{x_1}}{MU_{x_2}} = -\frac{2\frac{1}{x_1}}{3\frac{1}{x_2}} = -\frac{2x_2}{3x_1}$$

   These calculations show that $MRS_u = MRS_v$ and therefore, we can conclude that u and v have the same indifference curves.

2. It must be mentioned that preference rankings are ordinal and not cardinal. Therefore, many utility functions can correspond to a particular preference map. So, we can transform a utility function into an unlimited number of other utility functions that are consistent with the ordering of the individual. In our example, $v(x_1, x_2)$ is just a transformation of $u(x_1, x_2)$:

$$ln(u(x_1, x_2)) = ln(x^{0.2}, x^{0.3}) = lnx_1^{0.2} + lnx_2^{0.3} = 0.2lnx_1 + 0.3lnx_2 =$$
$$2ln(x_1) + 3ln(x_2) = v(x_1, x_2)$$

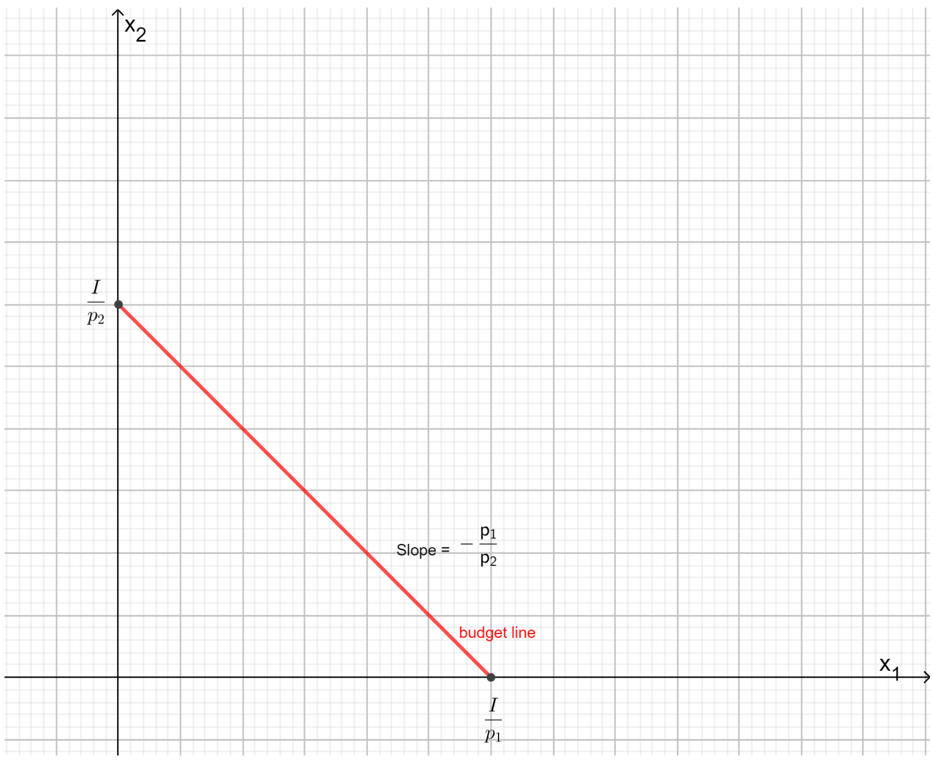
   Since $v(x_1, x_2)$ is just a transformation of $u(x_1, x_2)$, it is still consistent with his initial ordering and we can say, that u and v have the same indifference curves.

Problem 9

1. The Budget Constraint of Modena residents' is:

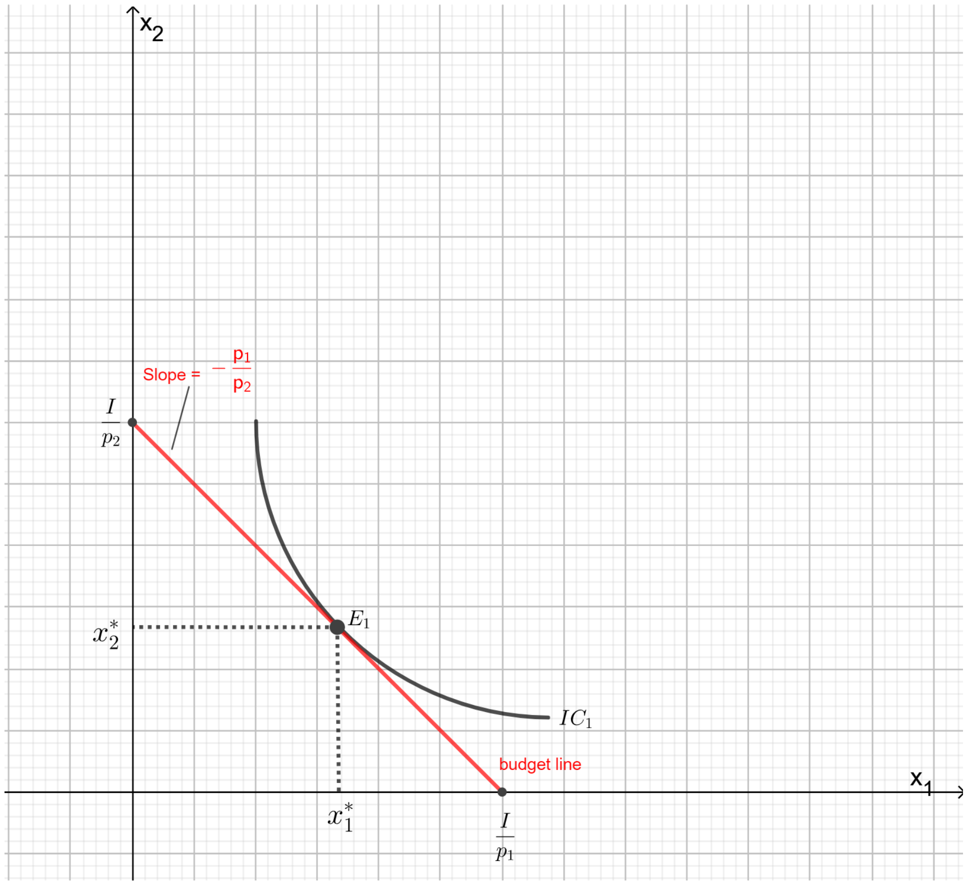
$$I = x_1 * p_1 + x_2 * p_2$$
$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2}$$

$\frac{I}{p_2}$ is the point of intersection of the budget line and the y-axis.$-\frac{p_1}{p_2}$ represents the slope of the budget line. When we set $x_2 = 0$ we will find the point of intersection of the budget line and the x-axis, which is $x_1 = \frac{I}{p_1}$. These findings are represented in the Figure below.

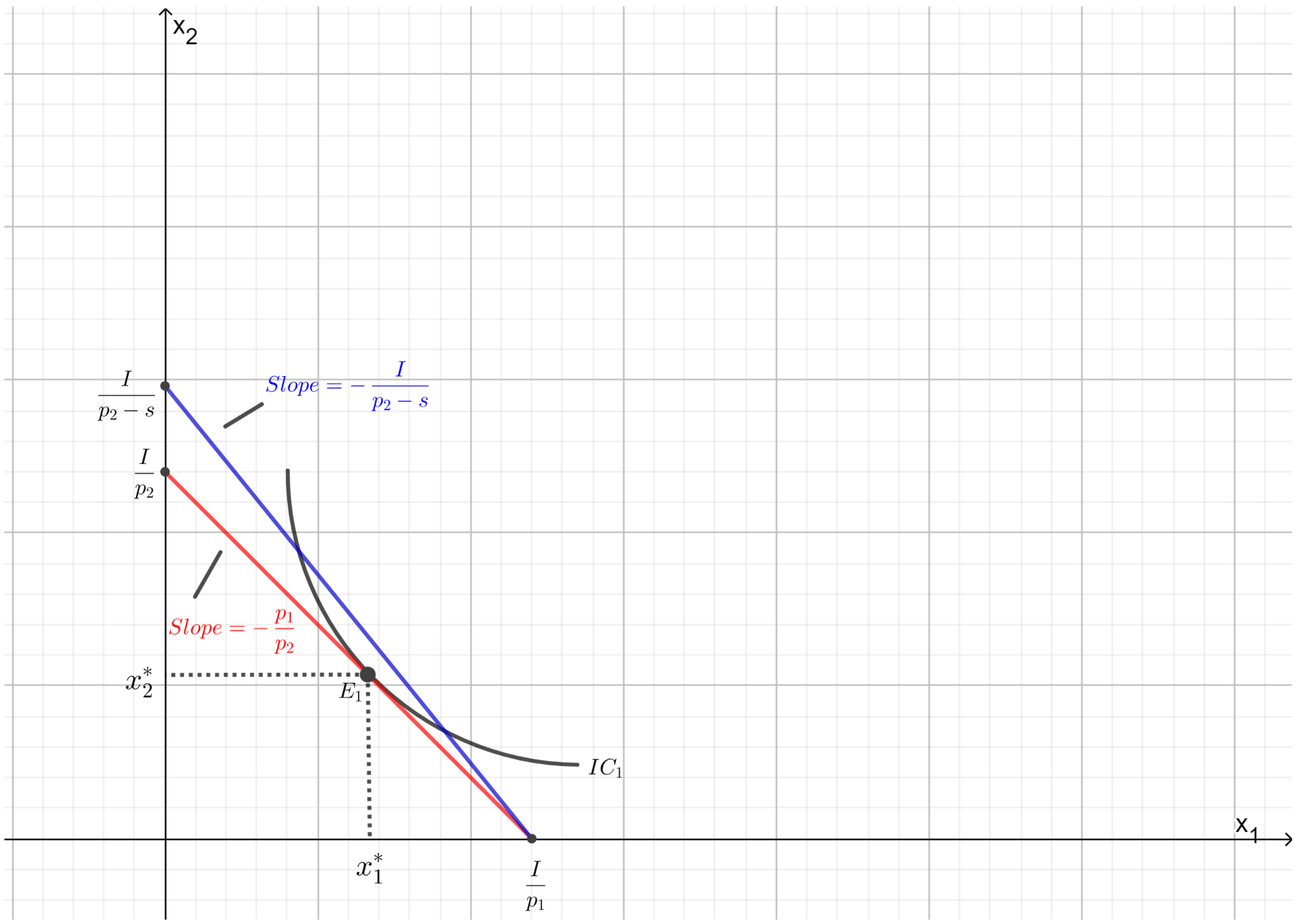


2. Since the utility function is a Cobb-Douglas Function, the total utility would be 0 if the individual would only consume one good. Therefore we know that we must find an interior solution. In this case, the optimal bundle must on the budget line and be on an indifference curve that does not cross it. That means, that the optimum bundle lies on the highest indifference curve that touches the budget line. In the point, where the two lines touch, the budget constraint and

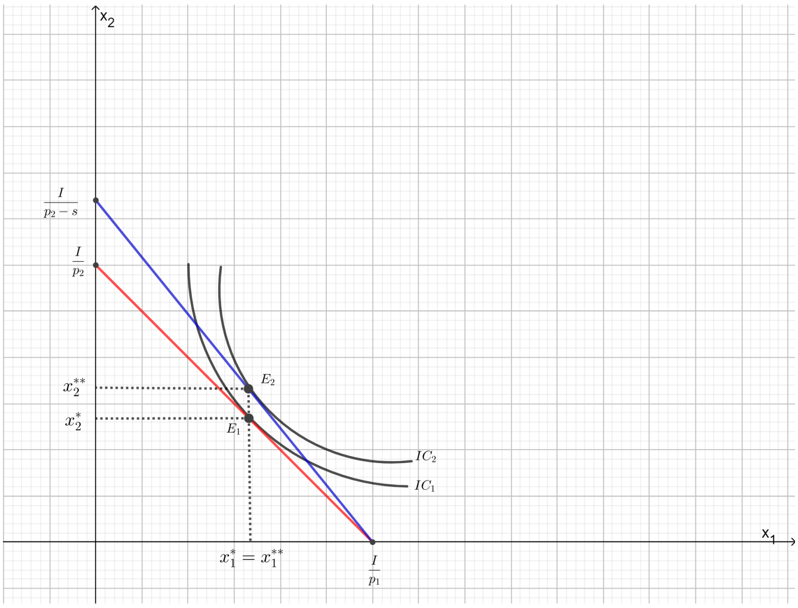
the indifference curve have the same slope, so that $MRS = MRT$. The following Figure shows this optimal bundle E_1.



3. Due to the subsidy, p_2 decreases and the residents can afford more tickets for philosophy events with the same income. Therefore the new budget line intercepts the y-axis at point $\frac{I}{p_2-s}$.The new budget line intercepts the x-axis at the same point $\frac{I}{x_1}$ like the old budget line since the price for x_1 did not change. The slope of the new budget line is $-\frac{p_1}{p_2-s}$. Since $p_2 - s < p_s$ we know that $|\frac{p_1}{p_2-s}| > |\frac{p_1}{p_2}|$. This means, that the slope of the new budget line is steeper.

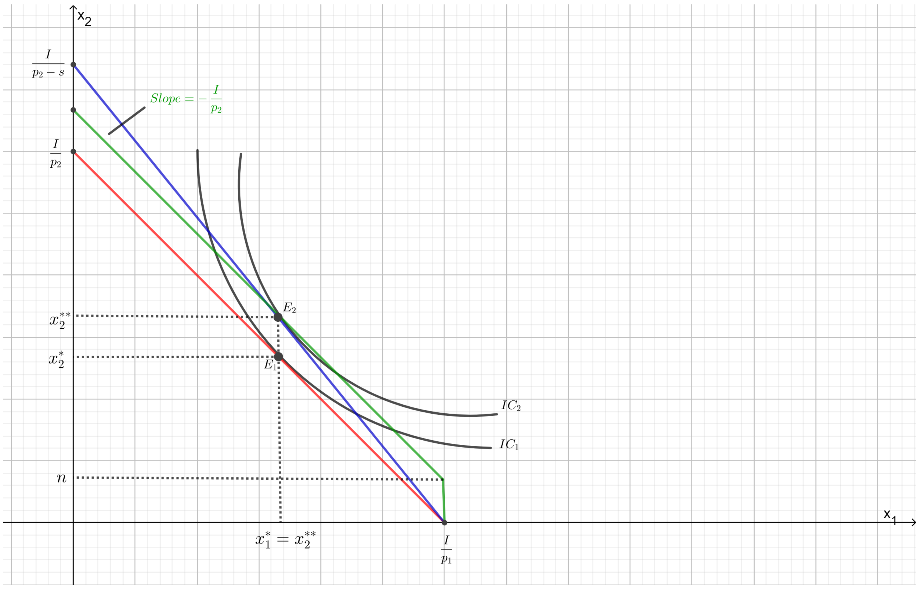


4. By moving from IC_1 to IC_2, the residents can increase their utility and the new optimal bundle is in point E_2.



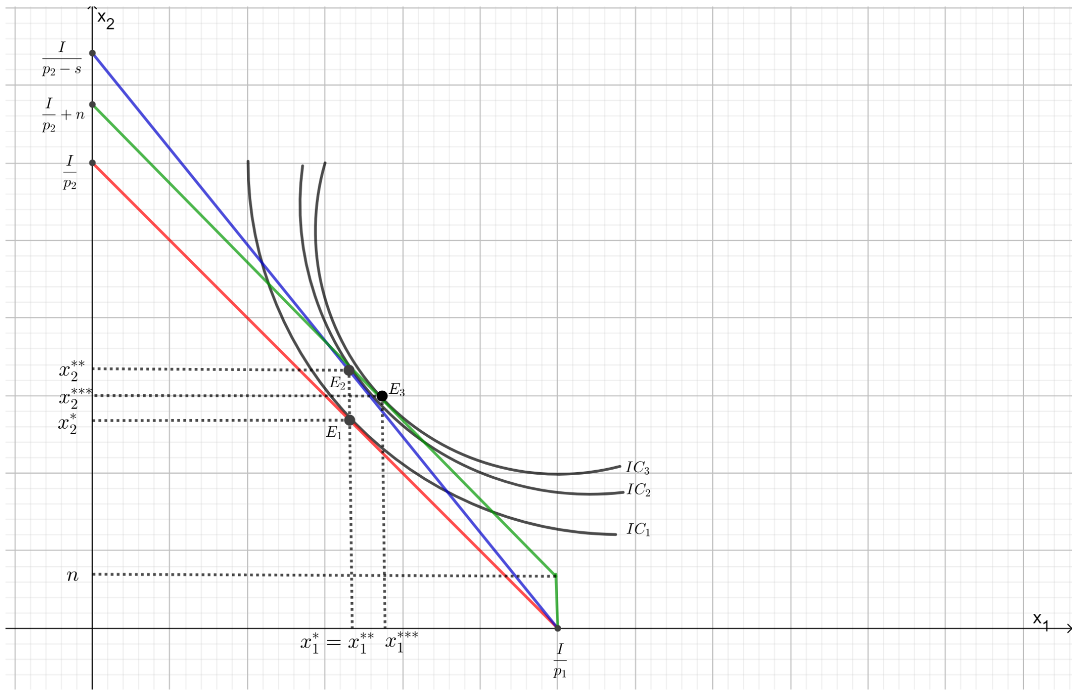
5. Since the utility function is a Cobb-Douglas Function, the quantity of concert tickets does not change (a change of p_2 does not affect x_1). Therefore $x_1^* = x_2^{**}$. So, only the quantity of tickets for philosophy events rise to the point x_2^{**}. In other words: The residents buy the same amount of concert tickets but more tickets for philosophy events.

6. With the new proposal of the other city council, the residents will always have at least $x_2^{**} - x_2^* = n$ philosophy tickets - even if they spend all their income on concert tickets. This leads to a kink in the new budget line.



7. Since the prices are unchanged, the new budget line will have the same slope as the original budget line. This is also the reason, why the new budget line is parallel to the original budget line.

8. The new budget line will lead to a new optimal bundle. The following figure will show, that x_1^{***} is greater than $x_1^{**} = x_1^*$. The reason is, that the residents get n tickets for philosophy events for free. The Cobb-Douglas utility function tells us, that the residents like to consume both goods together. Therefore they will use the extra resources they get from the government not only for philosophy tickets, but also for concert tickets. This behaviour is also the reason why x_2^{***} will be less than x_2^{**}. They will never use the extra resources to only increase the consumption of one good. The following figure shows the new optimal bundle $x^{***} = (x_1^{***}, x_2^{***})$



9. As already mentioned before, the residents will use the extra resources for consuming more of both goods. Therefore $x_1^{***} > x_1^*$ and $x2^{***} > x_2^*$.

10. The discussion above and also the figures showed, that the first policy should be implemented to maximize the residence' attendance to philosophy events. There, the price of the tickets for philosophy events is reduced and so the residents can't reallocate the extra resources like they would do, if the second policy was implemented. The figure under point 6 shows clearly, that $x_2^{**} > x_2^{***} > x_2^*$.

11. By the residents, the second policy is preferred. The reason is, that the optimal bundle lies on the highest indifference curve IC_3.