Microeconomics
Problem Set 2

Bocconi University Group 30

due: October 2, 2019 at 8.00 pm

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This problem set refers to Chapter 4 and Chapter 5.

Problem 1

1. The following figure shows three indifference curves:

2. Let's first calculate the MRS:

$$|MRS| = \frac{MU_{x_1}}{MU_{x_2}} = 2\frac{x_2}{x_1}$$

Now we can use the tangency condition and the budget constraint to find the demand function:

$$\begin{cases} |MRS| = 2\frac{x_2}{x_1} = \frac{p_1}{p_2} = |MRT| \\ p_1x_1 + p_2x_2 = I \end{cases}$$

$$\begin{cases} 2x_2p_2 = x_1p_1 \\ p_1x_1 = I - p_2x_2 \end{cases}$$

$$\begin{cases} x_1 = \frac{2I}{3p_1} \\ x_2 = \frac{I}{3p_2} \end{cases}$$

Therefore:

$$Q_1 = \frac{2I}{3p_1}$$

$$Q_2 = \frac{I}{3p_2}$$

3. We can calculate the cross-price elasticity between books and concert tickets with the following formula:

$$\epsilon^{x_2}_{p_1} = \frac{\partial Q_2}{\partial p_1}\frac{p_1}{Q_2}$$

But, we can't $\frac{\partial Q_2}{\partial p_1}$ because the demand for concert tickets does not depend on the price of books (the demand for concert tickets does only depend on I and p_2. The demand for books does only depend on I and p_1). Therefore, the cross-price elasticity $\epsilon^{x_2}_{p_1}$ is zero.

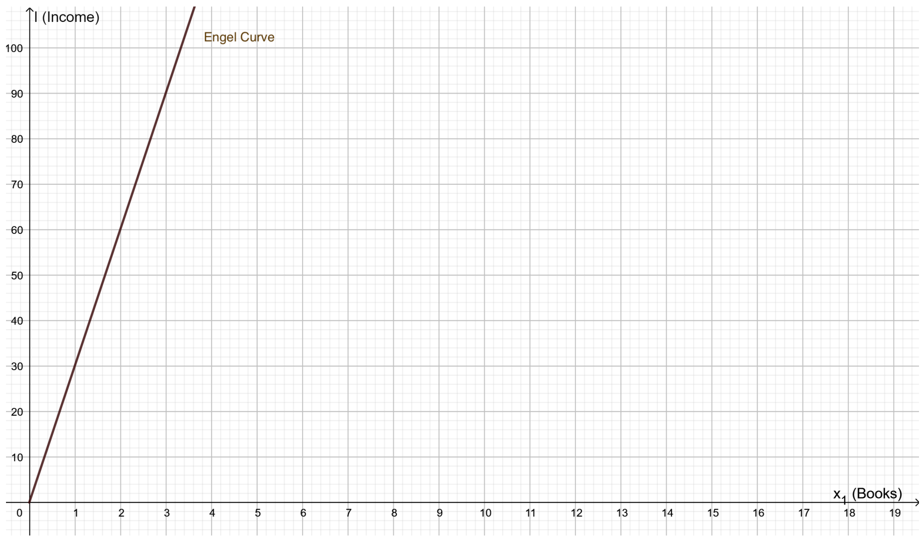
4. As already mentioned above: the demand for concert tickets does only depend on I and p_2. The demand for books does only depend on I and p_1. Therefore, an increase in the price of books does not impact the demand for concert tickets. Since the cross-price elasticity of demand is zero, concert tickets and books are uncorrelated goods.

5. The Engel curve shows the relationship between income and quantity demanded. To derive the Engel curve for books, we can just substitute $p_1 = 20$ into the demand function for books and then solve for I:

$$x_1 = \frac{2I}{3p_1}$$
$$x_1 = \frac{2I}{3*20}$$
$$I = 30x_1$$

Therefore, the Engel curve is $I = 30x_1$.

6. The following figure shows the Engel curve for books:



7. The income elasticity of the demand of books can be calculated with the following formula:

$$\xi_{x_1} = \frac{\partial Q_1}{\partial I} \cdot \frac{I}{Q_1}$$

$$\xi_{x_1} = \frac{2}{3p_1} \cdot \frac{I}{\frac{2I}{3p_1}}$$

$$\xi_{x_1} = 1$$

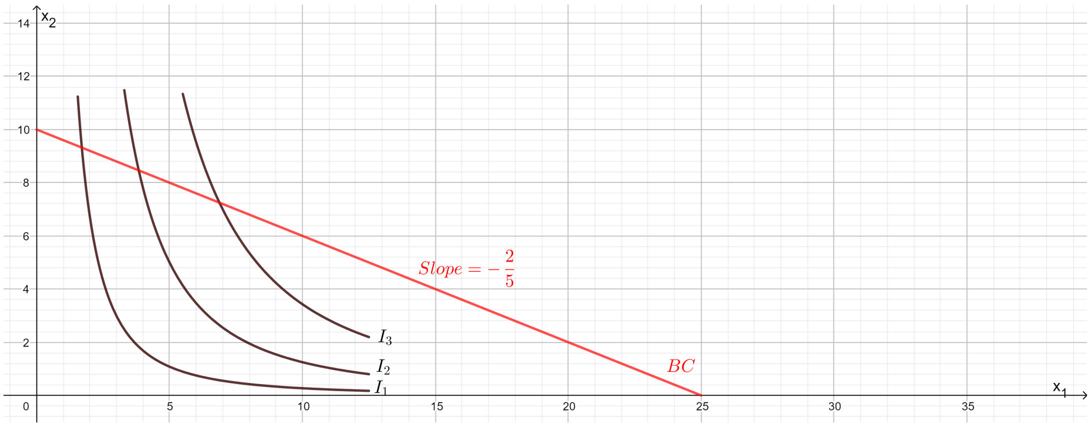
A normal good is a commodity for which more is demanded as income rises. A good is a normal good if its income elasticity is greater than or equal to zero: $\xi \geq 0$. Therefore, books are in our case normal goods. But, there are two types of normal goods. When $\xi > 1$ we say it's a luxury good. When $0 \leq \xi \leq 1$ we say it's a necessity. Since $\xi_{x_1} = 1$, in our case books are a necessity.

8. The budget constraint is:

$$I = p_1 x_1 + p_2 x_2$$

$$500 = 20x_1 + 50x_2$$

9. The following figure shows the budget constraint:



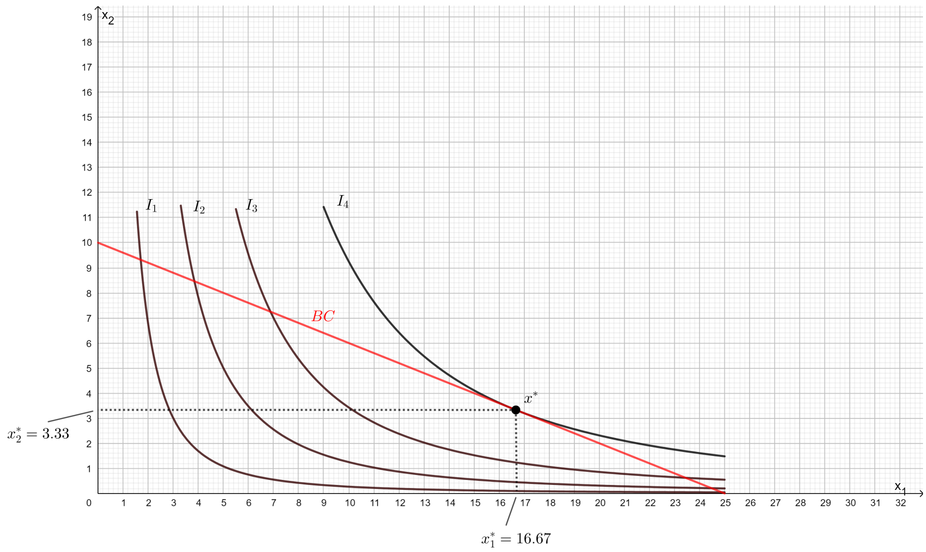
10. By substituting the value of the gift card and the prices into our demand functions, we get the optimal bundle:

$$x_1^* = \frac{2}{3} \cdot \frac{500}{20} = 16.67$$

$$x_2^* = \frac{500}{3 \cdot 50} = 3.33$$

Therefore, the optimal bundle x^* is $(16.67, 3.33)$.

11. Let's show the optimal bundle in our graph:



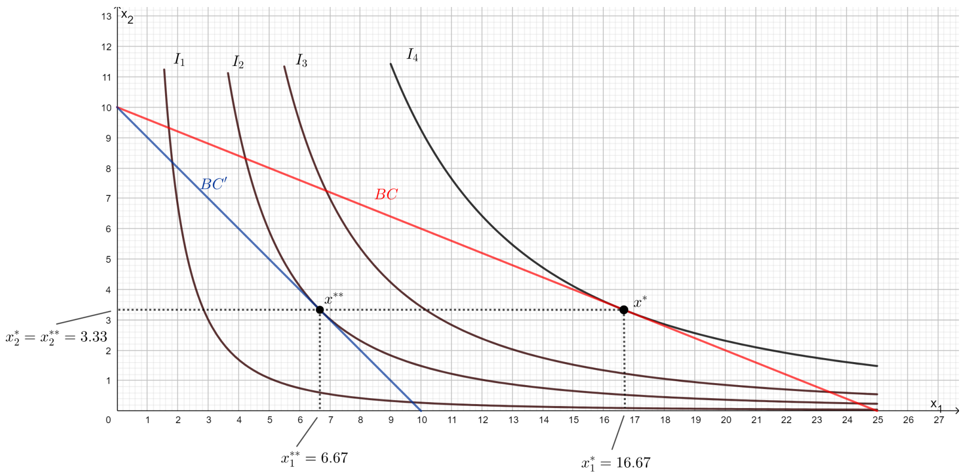
12. Since the demand functions did not change, we can again substitute the value of the gift card, the new p_1 and p_2 into the demand functions:

$$x_1^{**} = \frac{2}{3} \cdot \frac{500}{50} = 6.67$$

$$x_2^{**} = \frac{500}{3 \cdot 50} = 3.33$$

Therefore, the optimal bundle x^{**} is $(6.67, 3.33)$.

13. The following graph shows the new budget line BC' and the new optimal bundle x^{**}:



14. To solve this problem, we first have to derive the expenditure function E. To do so, we calculate the utility $\overline{U}$ Giorgia received with the original x_1 before the price change:

$$\overline{U} = 16.67^{\frac{2}{3}} \cdot 3.33^{\frac{1}{3}} = 9.74$$

Now, we have to find the least expenditure that is necessary to achieve this level of utility given the new prices. Therefore, we can replace in our utility function x_1 and x_2 with the demands $x_1 = \frac{2I}{3p_1}$ and $x_2 = \frac{I}{3p_2}$ respectively. Formally we also have to replace I with E. Then, we can solve our equation for E:

$$\overline{U} = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$$

$$\overline{U} = (\tfrac{2E}{3p_1})^{\frac{2}{3}} (\tfrac{E}{3p_2})^{\frac{1}{3}}$$

$$\overline{U} = E(\tfrac{2}{3p_1})^{\frac{2}{3}} (\tfrac{1}{3p_2})^{\frac{1}{3}}$$

$$E = \overline{U}(\tfrac{3p_1}{2})^{\frac{2}{3}} (3p_2)^{\frac{1}{3}}$$

Now we can substitute the new $p_1 = 50$, $p_2 = 50$ and $\overline{U} = 9.74$ and we get:

$$E = 920.37$$

Little remark: We used a rounded $\overline{U}$ in our calculation. When we would substitute the exact $\overline{U}$ we would receive $E = 921.008$. For solving the next exercises in this problem set, we will always use $E = 920.37$.

15. We already derived the expenditure function in number 14. So let's do it the same way again:

$$\overline{U} = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$$

$$\overline{U} = (\tfrac{2E}{3p_1})^{\frac{2}{3}} (\tfrac{E}{3p_2})^{\frac{1}{3}}$$

$$\overline{U} = E(\tfrac{2}{3p_1})^{\frac{2}{3}} (\tfrac{1}{3p_2})^{\frac{1}{3}}$$

$$E = \overline{U}(\tfrac{3p_1}{2})^{\frac{2}{3}} (3p_2)^{\frac{1}{3}}$$

Now we can substitute the new $p_1 = 50$, $p_2 = 50$ and $\overline{U} = 9.74$ (we calculated this in number 14) and we get:

$$E = 920.37$$

16. To get the optimal bundle, we now have to substitute $E = 920.37$ for I, 50 for p_1 and 50 for p_2 into our demand functions, we derived in number 2:

$$x_1^{***} = \tfrac{2}{3} \cdot \tfrac{920.37}{50} = 12.27$$

$$x_2^{***} = \tfrac{920.37}{3 \cdot 50} = 6.14$$

Therefore, the optimal bundle x^{***} is $(12.27, 6.14)$.

17. To find H_1, we have to differentiate the expenditure function E with respect to the price of good one:

$$H_1 = \frac{\partial E}{\partial p_1} = \overline{U}(1.5p_1)^{-\frac{1}{3}}(3p_2)^{\frac{1}{3}}$$

Simplify a little bit and we get:

$$H_1 = \overline{U}(\frac{2p_2}{p_1})^{\frac{1}{3}}$$

To find H_2, we have to differentiate the expenditure function E with respect to the price of good two:

$$H_2 = \frac{\partial E}{\partial p_2} = \overline{U}(1.5p_1)^{\frac{2}{3}}(3p_2)^{-\frac{2}{3}}$$

Simplify a little bit and we get:

$$H_2 = \overline{U}(\frac{p_1}{2p_2})^{\frac{2}{3}}$$

We could now again substitute $\overline{U} = 9.74, p_1 = 50, p_2 = 50$ into H_1 and H_2 and we would get exactly the same results for the new optimal bundle x^{***} as in number 16:

$$H_1 = 9.74(\frac{100}{50})^{\frac{1}{3}} = 12.27$$

$$H_2 = 9.74(\frac{50}{100})^{\frac{2}{3}} = 6.14$$

18. Let's analyze the substitution and income effect for books: First we can find the total effect of the increase in the price of books by calculating the difference between the number of books after the price change, x_1^{**}, and the number of books before the price change, x_1^* :

$$\text{Total effect} = 6.67 - 16.67 = -10$$

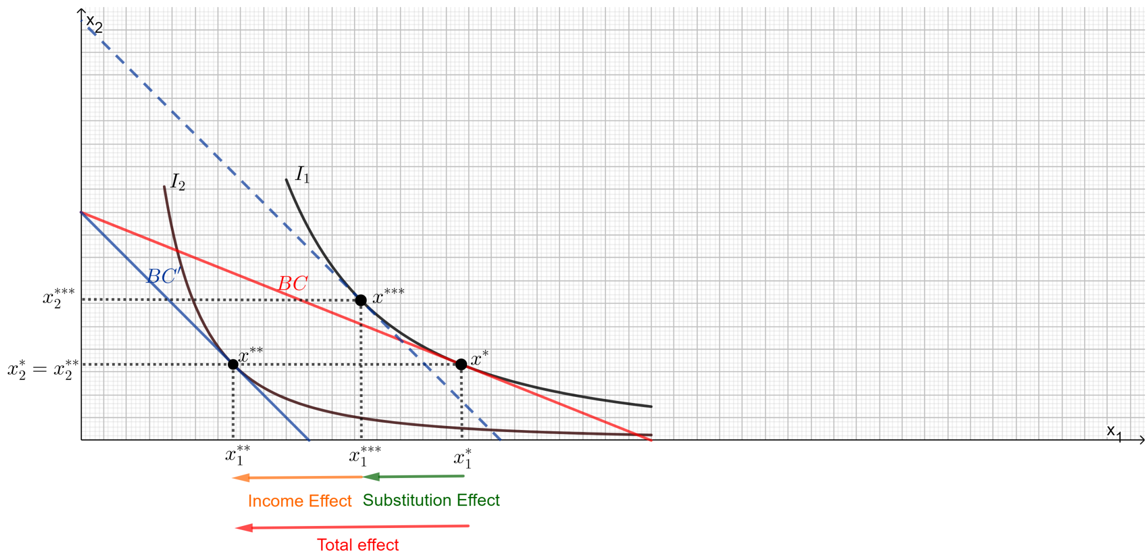
This total effect can be decomposed into the substitution and income effect. The compensated demand curve, which we derived in number 17, shows how the quantity demanded changes as the price rises, holding utility constant, so that the change in the quantity demanded reflects only the substitution effect from a price change. Therefore, we can find the substitution effect by calculating the difference between $H_1 = x_1^{***}$ (we calculated this in number 17) and the number of books consumed before the price change, x_1^*:

$$\text{substitution effect} = 12.27 - 16.67 = -4.4$$

Now, the income effect is the difference between the total effect and the substitution effect:

$$\text{Income effect} = -10 - (-4.45) = -5.6$$

19. The following graph shows the total effect, the income effect and the substitution effect caused by the increase in p_1:

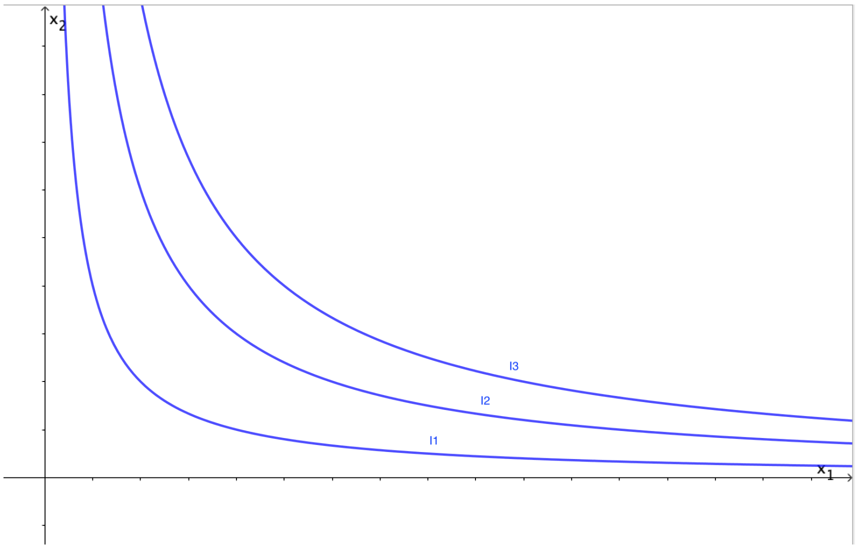


Problem 2

1. We can show, that $U(x_1, x_2)$ is just a transformation of a Cobb-Douglas utility function:

$$U(x_1, x_2) = \frac{1}{2}lnx_1 + \frac{1}{2}lnx_2 = (lnx_1)^{\frac{1}{2}} + (lnx_2)^{\frac{1}{2}} = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

Therefore, $\frac{1}{2}lnx_1 + \frac{1}{2}lnx_2$ has the same indifference curves as $x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$:

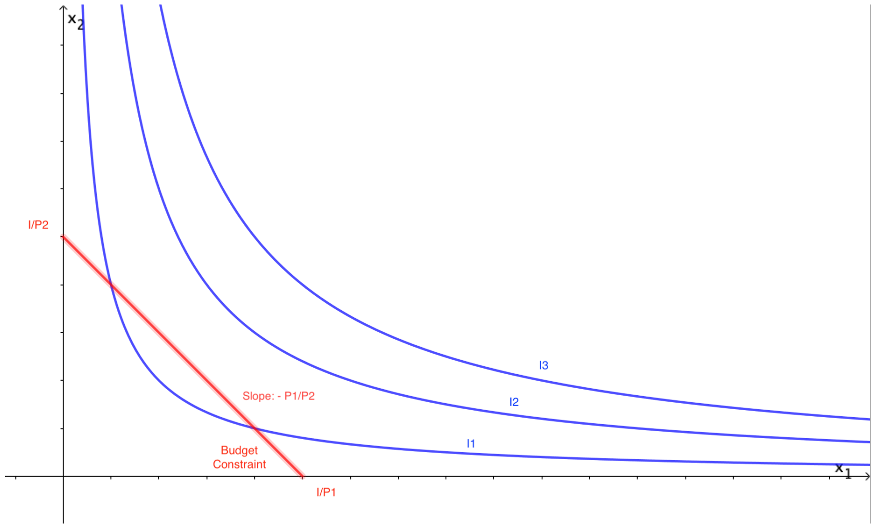


2. Jakob's MRS is:

$$|MRS| = \frac{MUx_1}{MUx_2} = \frac{\frac{1}{2}\frac{1}{x_1}}{\frac{1}{2}\frac{1}{x_2}} = \frac{x_2}{x_1}$$

3. In economics, the marginal rate of substitution (MRS) is the rate at which a consumer can give up some amount of one good in exchange for another good while maintaining the same level of utility. It can be derived as shown above. This means: Jakob is willing to give up $\frac{x_2}{x_1}$ units of Florentine Steaks, x_2, in order to get one additional unit of pici, x_1, and to remain equally happy.

4. The following graph shows Jakob's budget constraint:



5. The budget constraint indicates on the x-axis the amount of good x_1 and on the y-axis the amount of good x_2 that Jakob could afford if he's willing to spend all his income on of the goods. Therefore, the budget constraint shows the bundles of goods that can be bought if a consumer's entire budget is spent on these goods at given prices.The slope of this line indicates the amount of good x_2 Jakob has to give up to afford one more of good x_1, therefore it is the price ratio.

6. We can derive analytically Jakob's demand functions for pici and steaks, Q_1 and Q_2, in the following way:

$$\begin{cases} |MRS| = \frac{x_2}{x_1} = \frac{p_1}{p_2} = |MRT| \\ I = p_1 x_1 + p_2 x_2 \end{cases}$$

$$\begin{cases} p_1 x_1 = p_2 x_2 \\ I = p_1 x_1 + p_2 x_2 \end{cases}$$

$$\begin{cases} x_1 = \frac{1}{2} \frac{I}{p_1} \\ x_2 = \frac{1}{2} \frac{I}{p_2} \end{cases}$$

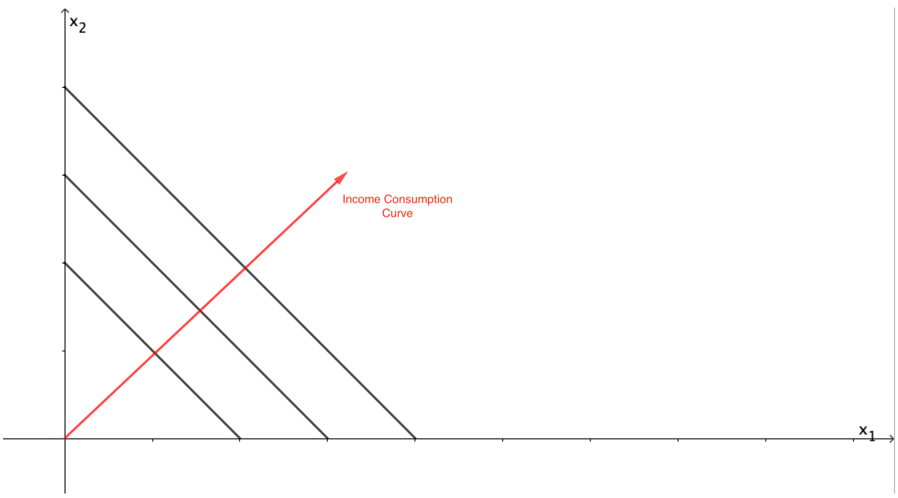
7. The income elasticity measure the responsiveness of the quantity demanded for a good or service to a change in income. Therefore it can be derived as follows:

$$\varepsilon = \frac{\partial x_1}{\partial I}\frac{I}{x_1} = \frac{1}{2p_1}\frac{I}{x_1} = \frac{I}{2p_1\frac{I}{2p_1}} = 1$$

$$\varepsilon = \frac{\partial x_2}{\partial I}\frac{I}{x_2} = \frac{1}{2p_2}\frac{I}{x_2} = \frac{I}{2p_2\frac{I}{2p_2}} = 1$$

8. A good is a normal good if its income elasticity is greater than or equal to zero: $\xi \geq 0$. Therefore x_1 and x_2 are in our case normal goods, so goods that experience an increase in their demand due to a rise in consumers' incomes. But, there are two types of normal goods. When $\xi > 1$ we say it's a luxury good. When $0 \leq \xi \leq 1$ we say it's a necessity.From the results obtained above, it is possible to assert that x_1 and x_2 are necessities.

9. Since we have two normal goods, the Income-consumption curve will be upward sloping:



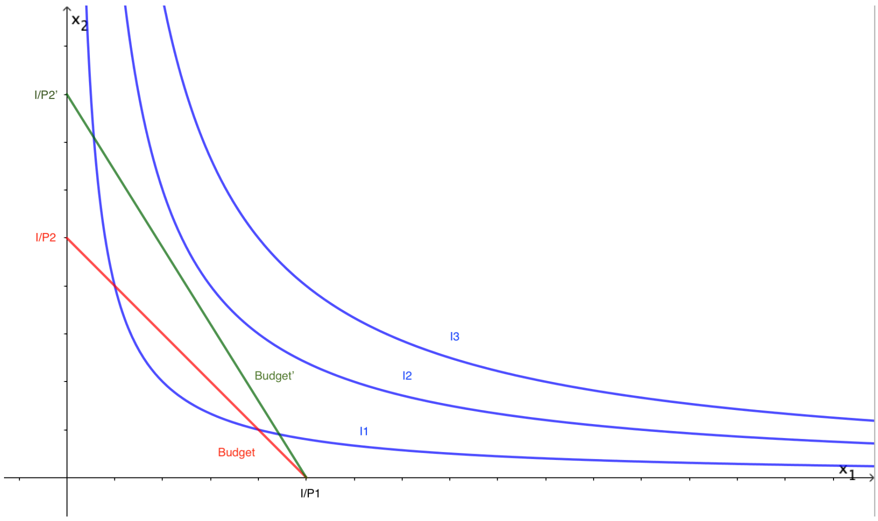
10. A decrease in the price of good x_2 ($p_2 \rightarrow p_2'$) determines a variation of the budget constraint, therefore we can evaluate it as follows:

$$\begin{cases} x_1 p_1 = x_2 p_2' \\ I = p_1 x_1 + p_2' x_2 \end{cases}$$

$$\begin{cases} x_1 = \frac{1}{2}\frac{I}{p_1} \\ x_2 = \frac{1}{2}\frac{I}{p_2'} \end{cases}$$

From the equation of good x_2 we can evaluate that a decrease in p_2' provokes an increase of $\frac{I}{p_2'}$, therefore the quantity demanded of good x_2 increases. We can also see, that the demand for x_1 does not depend on p_2. Therefore, the change in p_2 has no effect on the demand for good x_1.

11. The following graph shows the change in Jakob's budget constraint caused by the price change.



12. We can calculate the price elasticity of the demand function for steaks in the following way:

$$\varepsilon = \frac{\partial x_2}{\partial p_2}\frac{p_2}{x_2} = -\frac{I}{2(p_2)^2}\frac{p_2}{x_2} = -\frac{I}{2(p_2)^2}\frac{p_2}{\frac{I}{2(p_2)}} = -\frac{I}{2(p_2)^2}\frac{2(p_2)^2}{I} = -1$$

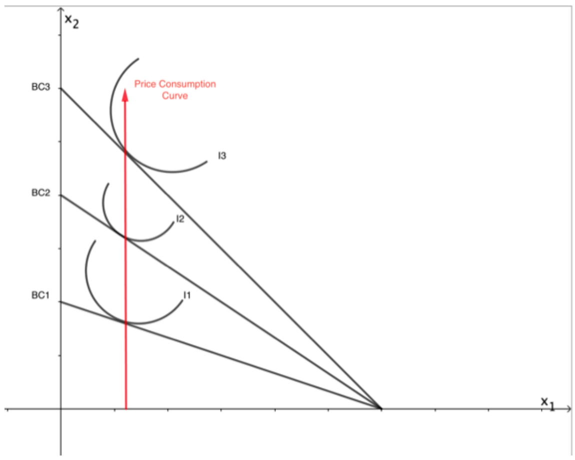
Therefore the price elasticity of demand is unitary, which means that an increase of 1% in price will lead to a decrease of 1% in the quantity demanded.

13. We calculate the cross-price elasticity of the demand function for pici, that is, $\varepsilon_{x_1}^{p_2}$ as follows:

$$\varepsilon_{x_1}^{p_2} = \frac{\partial x_1}{\partial p_2}\frac{p_2}{x_1} = 0$$

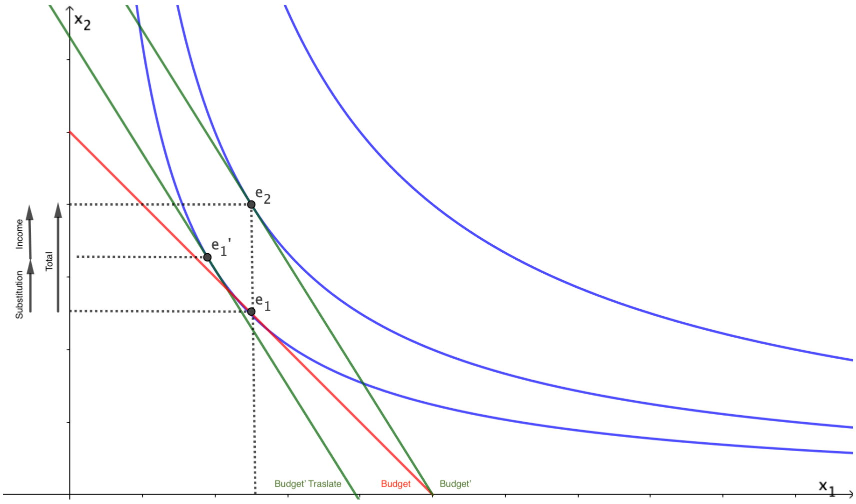
Therefore good x_1 and good x_2 are uncorrelated goods, because an increase in the price of good x_2, does not affect the quantity demanded of good x_1.

14. In number 13, we showed that good x_1 and good x_2 are uncorrelated goods. An increase in the price of good x_2, does not affect the quantity demanded of good x_1. Therefore, when the price of steaks changes, the price-consumption curve for pici is vertical.



15. The substitution effect is the change in the quantity demanded when the price for good x_2 increases, holding the price for x_1 and consumer's utility unchanged. The income effect shows the change in quantity demanded when income changes, holding prices unchanged. Good x_2 is a normal good. Therefore, the Substitution Effect and the Income Effect show in the same direction. This means that given a decrease in the price of good x_2, the amount of good x_2 demanded will increase by a quantity equal to the Total Effect = Substitution Effect + Income Effect. On the other hand, good x_1 is not influenced by the change in the price of good x_2 as they are unrelated goods.

16. The following graph shows the substitution and income effects. The red line is the original budget line. The new budget line is named "Budget'". The line "Budget'Translate" represents an imaginary budget constraint that is parallel to "Budget'" and tangent to the original indifference curve:



17. The compensated demand is obtained as $x_2 = H_{x_2}(p_1, p_2, \overline{U}) = \frac{\partial E}{\partial p_2}$, where E is the expenditure function. To get E, we start substituting $x_1 = \frac{E}{2p_1}$ and $x_2 = \frac{E}{2p_2}$ in the utility function (We have to remember that we can transform a utility function into an unlimited number of utility functions that are consistent with its initial ordering. As in number one already discussed, $U(x_1, x_2)$ is just a transformation of the Cobb-Douglas utility function $x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. Since this Cobb-Douglas Function is much easier to handle, we discussed to use this utility function instead of $U(x_1, x_2)$ to derive the expenditure function).

$$\overline{U} = (\tfrac{E}{2p_1})^{\frac{1}{2}}(\tfrac{E}{2p_2})^{\frac{1}{2}}$$
$$\overline{U} = E(\tfrac{1}{2p_1})^{\frac{1}{2}}(\tfrac{1}{2p_2})^{\frac{1}{2}}$$
$$E = \overline{U}(2p_1)^{\frac{1}{2}}(2p_2)^{\frac{1}{2}}$$

The compensated demand curve is:

$$x_2 = H_{x_2}(p_1, p_2, \overline{U}) = \frac{\partial E}{\partial p_2} = \overline{U}(\tfrac{p_1}{p_2})^{\frac{1}{2}} = \overline{U}(p_1)^{\frac{1}{2}}(p_2)^{-\frac{1}{2}}$$

Therefore, the substitution elasticity of the compensated demand $\varepsilon_{x_1}^{p_2}$ is:

$$\varepsilon_{x_1}^{p_2} = \frac{\partial Hx_2}{\partial p_2}\frac{p_2}{Hx_2} = -\frac{1}{2}$$

18. By using the Slutsky equation, we can decompose the substitution and income effects of the price change as follows:

$$\text{Total Effect } (\varepsilon) = \text{Substitution Effect } (\varepsilon^*) + \text{Income Effect } (\text{-}\theta\xi)$$

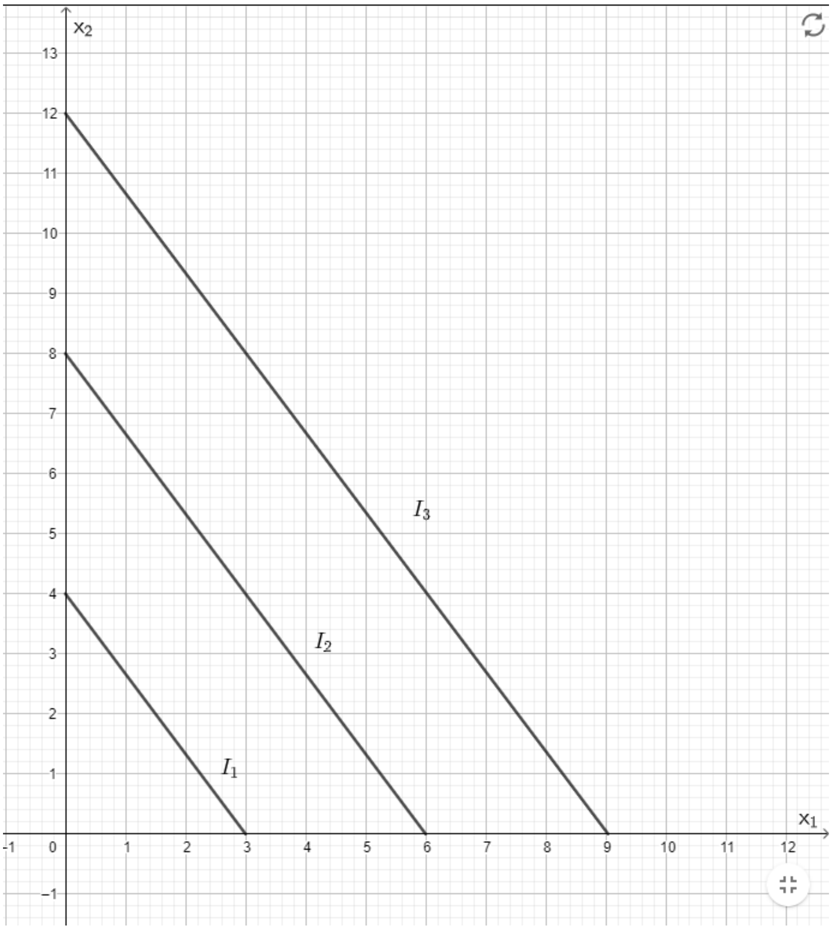
$$-1 = -\tfrac{1}{2} - (\tfrac{1}{2} \cdot 1)$$

Problem 3

1. The goods are perfect substitutes, so a possible utility function could be U(x_1,x_2) = ax_1 + bx_2, because each 3 cups of tea and 4 cups of coffee provide the same amount of utility (U). Therefore, when Hiromi buys 4 times x_1 and 3 times x_2, she receives utility level U. If a = 4 and b = 3, Hiromi's U function is:

$$U = 4x_1 + 3x_2$$

2. The following graph shows 3 indifference curves.



3. In the following Hiromi's MRS is calculated:

$$|MRS| = \frac{MU_{x_1}}{MU_{x_2}} = \frac{4}{3}$$

This shows that in order to obtain another unit of x_1, Hiromi is willing to give up around 1.33 units of x_2 and remain equally as "happy".

4. As the good are perfect substitutes we can have both a corner or an interior solution.
   By combining the tangency rule (MRS = MRT) with the budget constraint we get:

$$\begin{cases} |MRS| = |MRT| & \rightarrow \frac{4}{3} = \frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 = I \end{cases}$$

   Therefore, we know that:

$$3p_1 = 4p_2$$
$$p_1 = \tfrac{4}{3}p_2$$

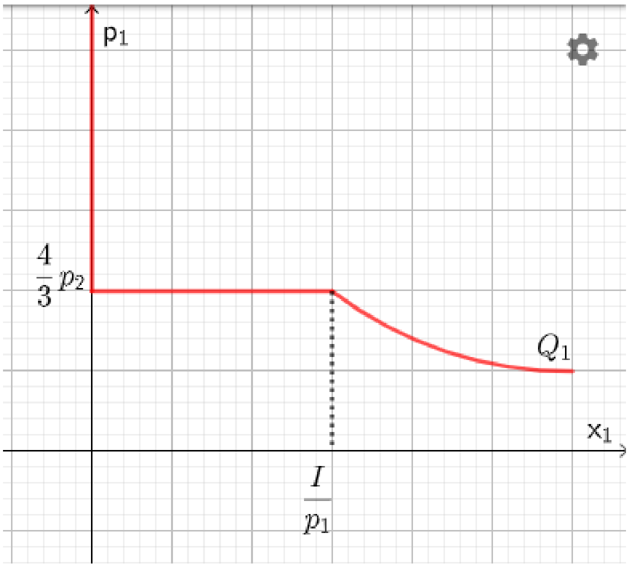
   Thus, when:

$$\begin{cases} p_1 < \frac{4}{3}p_2 & \textit{only } x_1 \textit{ consumed} \\ p_1 = \frac{4}{3}p_2 & \text{any combination of } x_1 \text{ and } x_2 \text{ is optimal} \\ p_1 > \frac{4}{3}p_2 & \textit{only } x_2 \textit{ consumed} \end{cases}$$

   With this knowledge, we can now formulate the demand function for tea, Q_1:

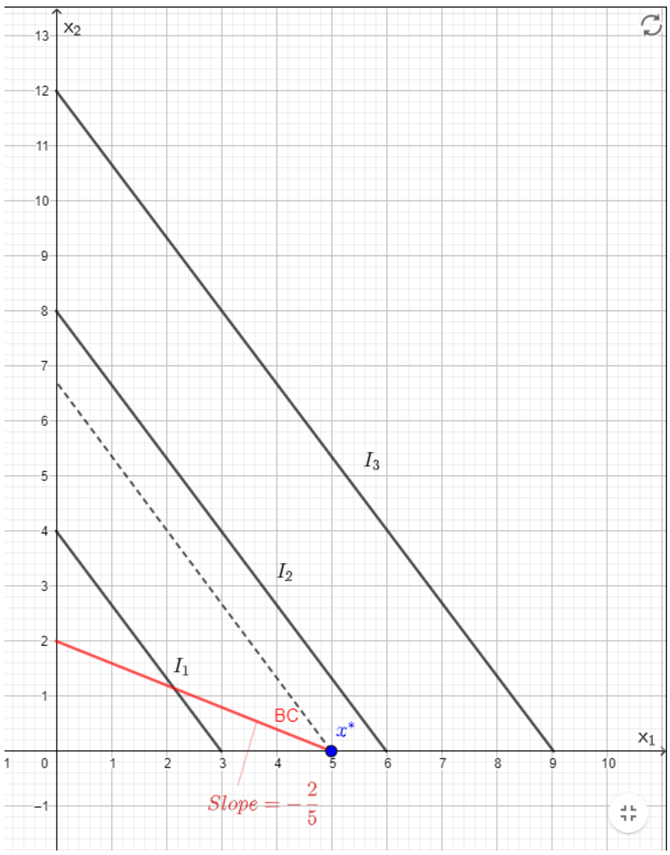
$$\begin{cases} x_1 = \frac{I}{p_1} & if \;\; p_1 < \frac{4}{3}p_2 \\ x_1 = 0 & if \;\; p_1 > \frac{4}{3}p_2 \\ x_1 \in (0, \frac{I}{p_1}) & if \;\; p_1 = \frac{4}{3}p_2 \end{cases}$$

5. The following graph shows the demand function Q_1, we derived in number 4



6. If $I = 10$, $p_1 = 2$ and $p_2 = 5$ the budget constraint is $2x_1 + 5x_2 = 10$.

7. The following graph shows the budget constraint:



8. There is only a corner solution, because:

$$MRS \neq \text{MRT}$$

$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{4}{3} \neq \frac{2}{5} = \frac{P_1}{p_2}$$

Comparing the utility per euro for tea and coffee, we get:

$$\begin{cases} \frac{MUx_1}{p_1} = \frac{4}{2} = 2 \\ \frac{MUx_2}{p_2} = \frac{3}{5} \end{cases}$$

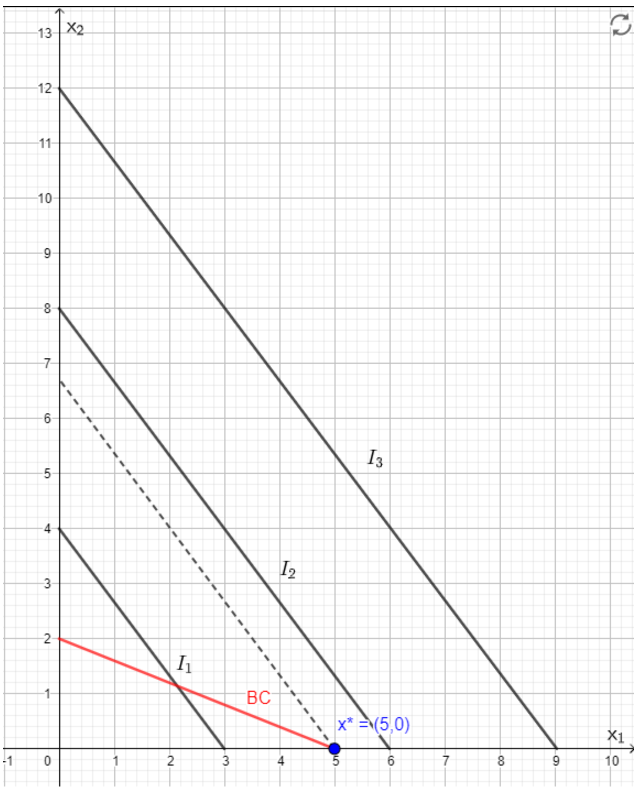
we see that:

$$\frac{MUx_1}{p_1} > \frac{MUx_2}{p_2} \rightarrow x_1 \text{ will be consumed (Corner solution)}$$

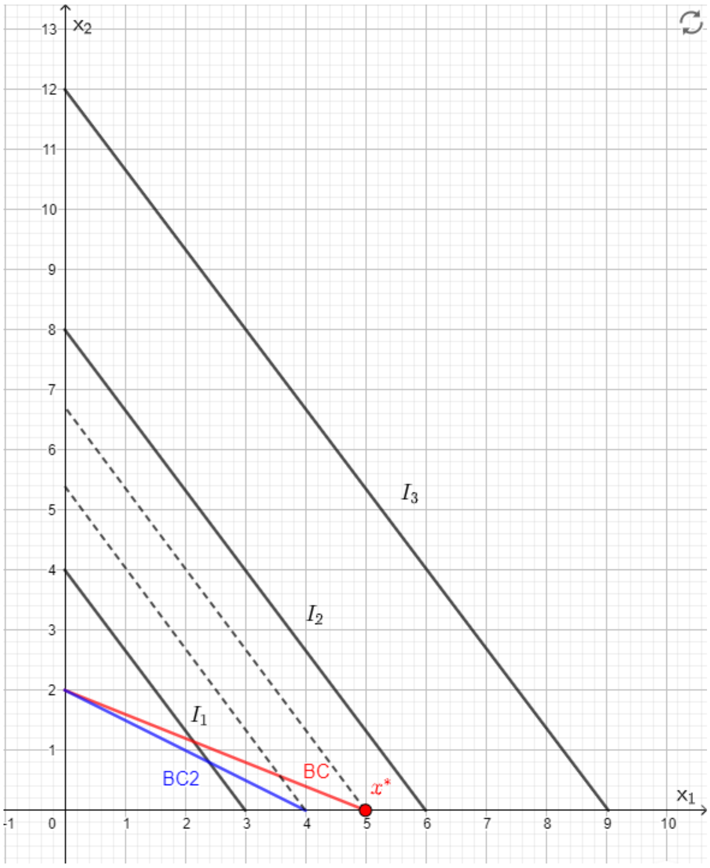
Therefore:

$$x^* = (5, 0)$$

9. The following graph shows the bundle x^*:



10. The new budget constraint $BC2$ is $2.5x_1$+$5x_2$=10



11. Because of

$$MRS \neq \text{MRT}$$

$$\frac{MU_{x_1}}{MU_{x_2}} = \frac{4}{3} \neq \frac{2.5}{5} = \frac{P_1}{p_2}$$

$$\rightarrow \text{corner solution}$$

Comparing the utility per euro for tea and coffee, we get:

$$\begin{cases} \frac{MUx_1}{p_1} = \frac{4}{2.5} = 1.6 \\ \frac{MUx_2}{p_2} = \frac{3}{5} = 0.6 \end{cases}$$

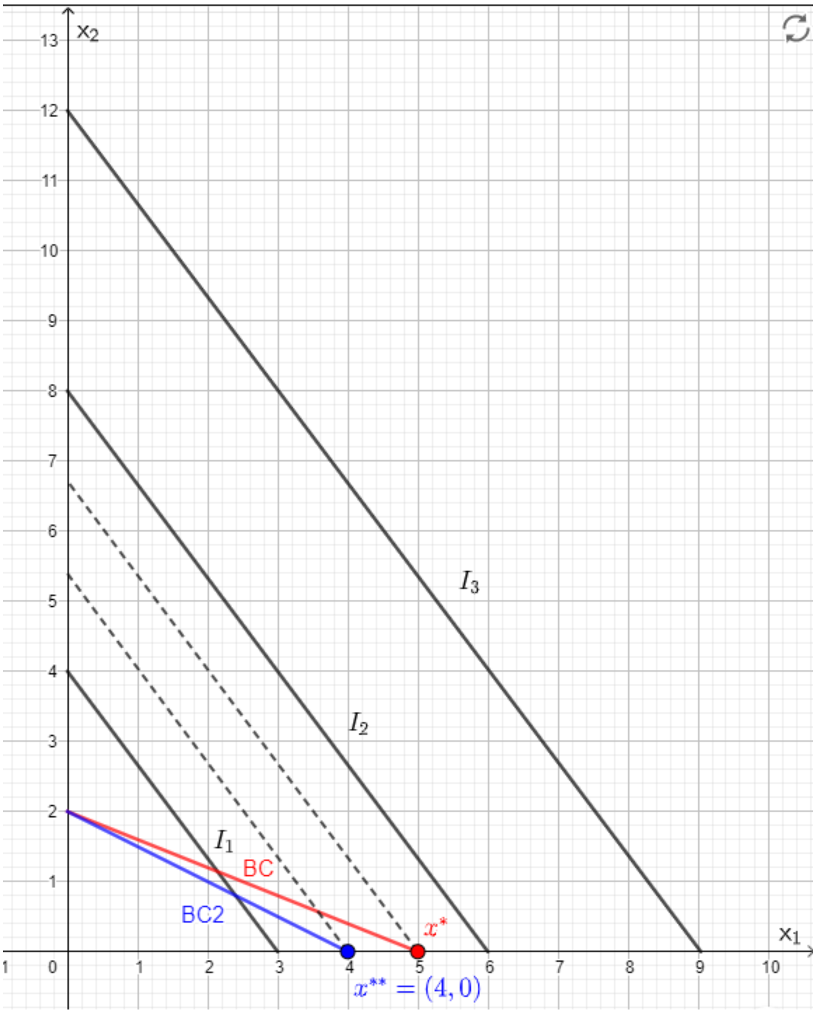
we see that:

$$\frac{MUx_1}{p_1} > \frac{MUx_2}{p_2} \rightarrow x_1 \text{ will be consumed (Corner solution)}$$

Therefore:

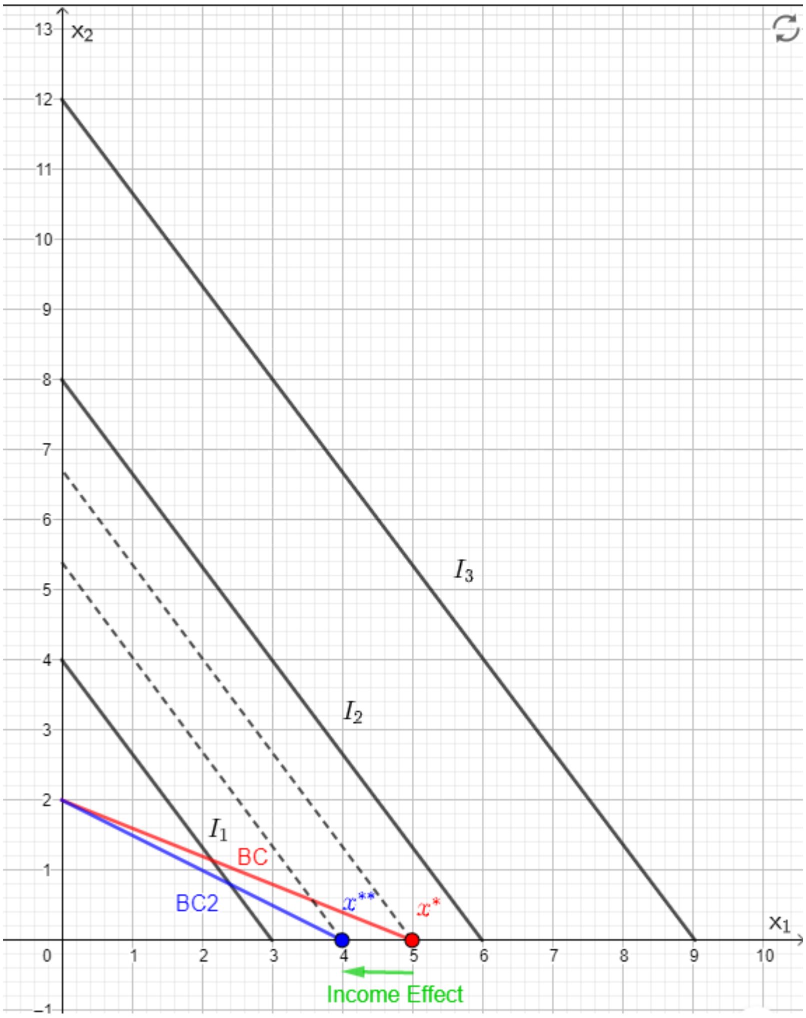
$$x^{**} = (4, 0)$$

12. The blue point represents the bundle $x^{**} = (4/0)$ in the following graph.



13. There is no substitution effect. Only the Income effect takes place. The increase in price p_1 of tea makes Hiromi relatively poorer, so she purchases less of x_1, pushing the new BC inwards.

14. The following graph shows only the income effect, since there is no substitution effect:



15. Analogically, we have again:

$$MRS \neq \text{MRT}$$

$$\frac{4}{3} \neq \frac{2}{2.5}$$

Therefore, we have again a corner solution. Comparing the utility per euro for tea and coffee, we get:

$$\begin{cases} \frac{MUx_1}{p_1} = \frac{4}{2} = 2 \\ \frac{MUx_2}{p_2} = \frac{3}{2.5} = 1.2 \end{cases}$$

We see that:

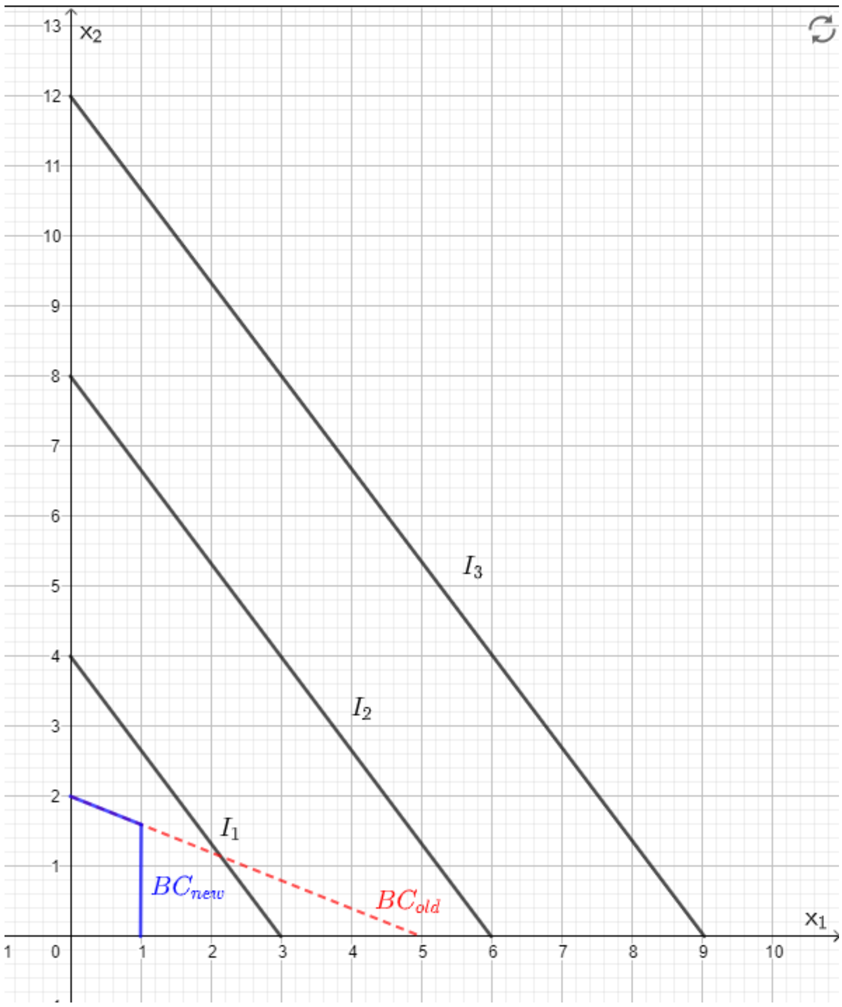
$$\frac{MUx_1}{p_1} > \frac{MUx_2}{p_2} \rightarrow x_1 \text{ will be consumed (Corner solution)}$$

Therefore: The optimum bundle remains the same as in part 8.

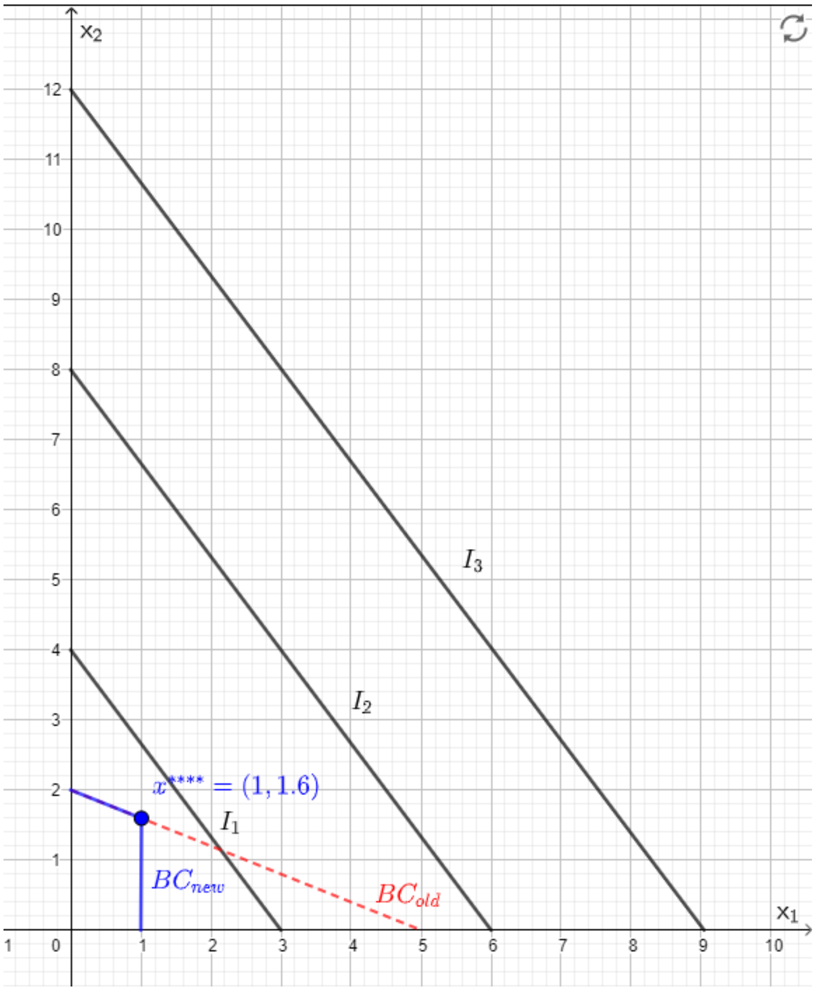
$$x^{***} = (5, 0)$$

16. Because Hiromi didn't change her consumption bundle (unlike in part 10) no income and substitution effects apply.

17. The BC curve after the shortage (BC_{new}) is kinked - she can only purchase no or 1 cup of tea. The upper part of the curve has the same slope as the initial BC, which is -$\frac{2}{5}$.



18. The following graph shows the new optimal consumption bundle x^{***}



With the new BC curve, —MRS— would still be bigger than —MRT—. Therefore, Hiromi would prefer to only consume good x_1 and she will buy as much tea as possible. Therefore, the new optimal quantity of x_1 must be 1. Because of the shortage, she can only buy this one cup of tea. To maximize utility, she will spend the other 8 Euros for coffee. Therefore, she can afford $\frac{8}{5}$ cups of coffee. Therefore, the optimal bundle x^{***} is $(1, 1.6)$

19. We showed above, that Hiromi should spend all her income for tea to maximize her utility. This is not possible because of the shortage. But, the utility function derived in number one shows, that she still gets some utility from drinking coffee. Therefore, she would spend all her money left (the 8 Euros) for coffee. But, despite the optimal consumption occurring at $x_1^{***} = 1$ and $x_2^{***} = 1.6$, she cannot buy a fraction of a cup of coffee, so Hiromi will buy only one cup of each.

This will leave $(10 - 2 - 5) = 3$€ of her I unspent.

20. In the following calculation, we assume like in number 19, that Hiromi can't buy a fraction of coffee (if she could, her $U_{present}$ would be 8.9 and not 7).

$$\begin{cases} U_{old} = 4 \cdot 5 + 0 = 20 \\ U_{present} = 4 \cdot 1 + 3 \cdot 1 = 7 \end{cases}$$

Therefore, her total utility has fallen.

Problem 4

1. x_1 and x_2 are **perfect complements** because for Zeno one activity (x_1) is only useful if it is consumed together with the other one (x_2). He always wants to consume 1 hour of martial arts with 5 hours of intensive private lessons of microeconomics.

   Perfect complements are goods that a consumer is interested to buy only in fixed proportions.

2. For perfect complements, the general expression for the utility function can be written in the following way:

$$U(x1, x2) = min(ix_1, jx_2)$$

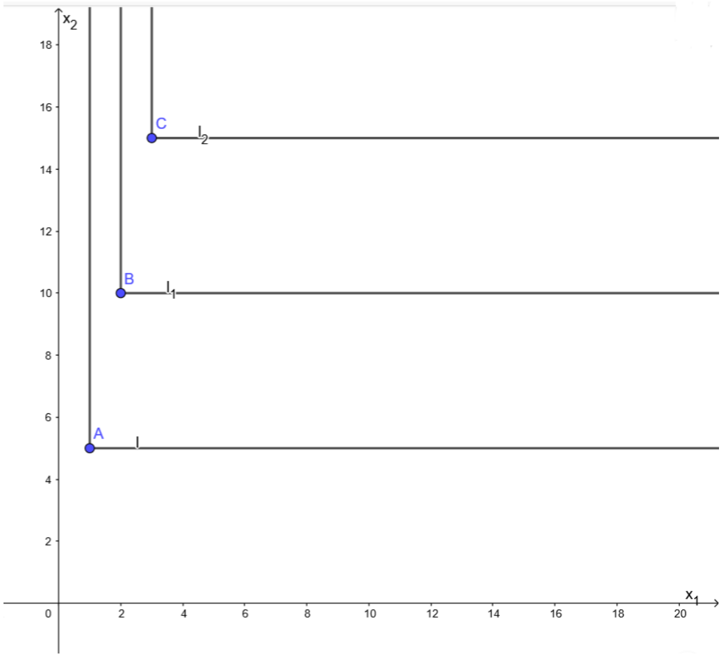
   One possible bundle for Zeno is for example $1x_1 and 5x_2$. Therefore, we know:

$$1i = 5j$$

   This equality must hold for any i and j; for example, if we choose $j = 1$, we then obtain that i must be equal to 5. Therefore, our utility function is:

$$U(x_1, x_2) = min(5x_1, x_2)$$

3. The following graph shows the three indifference curves:



4. Zeno's $MRS = 0$ because the utility function is **nondifferentiable**.
   $\rightarrow$ He's unwilling to substitute more of one good for less of another.

5. The demand functions Q_1 and Q_2 can be calculated as follows:

$$\begin{cases} 5x_1 = x_2 \rightarrow x_1 = \frac{1}{5}x_2 \\ p_1x_1 + p_2x_2 = I \end{cases}$$

   $p_1x_1 + p_2 5x_1 = I \rightarrow p_1x_1 + 5p_2x_1 = I \rightarrow x_1(p_1 + 5p_2) = I$
   $\rightarrow Q_1 = \frac{I}{p_1+5p_2}$
   $p_1\frac{1}{5}x_2 + p_2x_2 = I \rightarrow \frac{1}{5}p_1x_2 + p_2x_2 = I \rightarrow x_2(\frac{1}{5}p_1 + p_2) = I$
   $\rightarrow Q_2 = \frac{5I}{p_1+5p_2}$

6. Calculate own price and cross-price elasticities of the demand function for martial arts.
   Own price elasticity
   ϵ= percentage change in quantity demanded/percentage change in price.
   $\epsilon = \frac{\Delta Q_1}{\Delta p_1}\frac{p_1}{Q_1} = \frac{\partial Q_1}{\partial p_1}\frac{p_1}{Q} \rightarrow = -\frac{p_1}{p_1+5p_2}$

Cross price elasticity

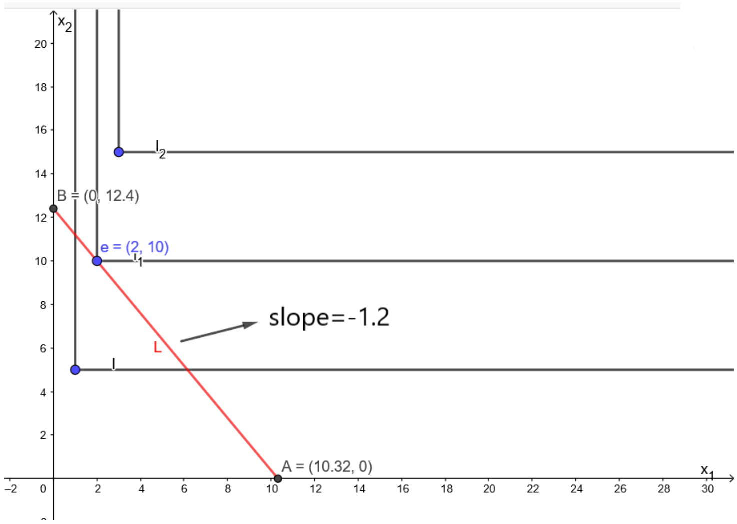
ϵ= percentage change in quantity demanded/percentage change in price of the other good.

$\epsilon = \frac{\Delta Q_1}{\Delta p_2}\frac{p_2}{Q_1} = \frac{\partial Q_1}{\partial p_2}\frac{p_2}{Q_1} \rightarrow = -\frac{5p_2}{p_1+5p_2}$

7. The cross-price elasticity is negative. That means that Zeno will consume less hours of martial arts, x_1 when the price for microeconomics lessons, p_2, increases. Therefore, the sign of the **cross price elasticity** is in line with our expectations because it proves that good x_1 and good x_2 are **complements**.

8. The budget constraint looks as follows: **Budeget constraint**$\rightarrow$ $p_1x_1 + p_2x_2 = I \rightarrow 30x_1 + 25x_2 = 310$

9. The following graph shows the budget constraint:



10. Zeno's optimal consumption bundle $x^* = (x_1^*, x_2^*)$ can be derived as follows:

$$x_1^* = \frac{I}{p_1 + 5p_2} = \frac{310}{155} = 2$$
$$x_2^* = \frac{5I}{p_1 + 5p_2} = \frac{1550}{155} = 10$$
$$x^* = (x_1^*, x_2^*) = (2, 10)$$

11. Point "e" in the graph in number 9 represents the optimal bundle $x^* = (x_1^*, x_2^*)$.

12. We can calculate the new optimal bundle $x^{**} = (x_1^{**}, x_2^{**})$ in the following way:

$$x_1^{**} = \frac{I}{p_1 + 5p_2^{'}} = \frac{310}{180} = 1.7$$

$$x_2^{**} = \frac{5I}{p_1 + 5p_2^{'}} = \frac{1550}{180} = 8.6$$

$$x^{**} = (x_1^{**}, x_2^{**}) = (1.7, 8.6)$$

13. No substitution effect occurs because Zeno is unwilling to substitute between good x_1 and good x_2.
Subsequently we only have an income effect that is equal to the total effect.

14. First, we have to find the expenditure function, E:
$x_1 = \frac{I}{p_1+5p_2}$ $\quad x_2 = \frac{5I}{p_1+5p_2}$
$U = min(5x_1, x_2) = min(\frac{E}{p_1+5p_2}, \frac{5E}{p_1+5p_2})$
$E = U\frac{p_1+5p_2}{5}$

Now we can find H_1 and H_2 by differentiating the expenditure function with respect to p_1 and p_2 respectively:

$\rightarrow H_1 = \frac{\partial E}{\partial p_1} = \frac{1}{5}U$
$\rightarrow H_2 = \frac{\partial E}{\partial p_2} = \frac{5}{5}U = U$

15. H_1 and H_2 do not depend on prices so when p_2 raises they do not change.

16. Initial Utility $U = min(5x_1, x_2)$ where $x_1 = 2$ and $x_2 = 10$
$\rightarrow U = min(10, 10) = 10$
$H_1 = \frac{1}{5}U = \frac{1}{5}10 = 2$
$H_2 = U = 10$

$$x^{***} = (x_1^{***}, x_2^{***}) = (2, 10)$$

17. The level of income he would need to afford x^{***} can be calculated as follows:

$$E = U\frac{p_1+5p_2}{5} = 10\frac{30+5 \cdot 30}{5} = 360$$

The level of income increases because the optimal bundle(x^{***}) remains the same as the initial optimal bundle(x^{*}) while p_2 raises. Subsequently the level of income has to raise so that Zeno will be able to afford the same but more expensive bundle.

Problem 5

1. Dmitrii's utility function is a quasilinear utility function, because it is linear for x_1 but it's not linear for x_2.

2. Since we have a quasilinear utility function, there may be an internal solution, but also a corner solution. We first check for an interior solution using the tangency condition and the budget constraint. If we find that these conditions imply that Dimitrii wants to buy positive quantities of both goods, we have found an interior solution. Otherwise we have to determine a corner solution.

$$|MRS| = \frac{MUx_1}{MUx_2} = \frac{3}{\frac{1}{x_2}} = 3x_2$$

$$\begin{cases} |MRS| = 3x_2 = \frac{p_1}{p_2} = |MRT| \\ I = p_1x_1 + p_2x_2 \end{cases}$$

$$\begin{cases} x_2 = \frac{1}{3}\frac{p_1}{p_2} \\ -p_1x_1 = p_2x_2 - I \end{cases}$$

$$\begin{cases} x_2 = \frac{1}{3}\frac{p_1}{p_2} \\ x_1 = \frac{I}{p_1} - \frac{1}{3} \end{cases}$$

We see, that x_1 is only positive if $\frac{I}{p_1} > \frac{1}{3}$. We have interior solutions only in these cases. So, we can make the following statements:

We have Interior Solutions:

$$\begin{cases} Q_2 = \frac{1}{3}\frac{p_1}{p_2} \\ Q_1 = \frac{I}{p_1} - \frac{1}{3} \\ \frac{I}{p_1} > \frac{1}{3} \end{cases}$$

Instead we have Corner Solutions:

$$\begin{cases} Q_2 = \frac{I}{p_2} \\ Q_1 = 0 \\ \frac{I}{p_1} \leq \frac{1}{3} \end{cases}$$

3. The condition for having an interior solution for the optimal consumption bundle is: $\frac{I}{p_1} > \frac{1}{3}$.

4. Calculate the income elasticity of Dmitrii's demand function for a tennis lesson, x_2. The income elasticity of Dmitrii's demand function for a tennis lesson, x_2, can be calculated with the following formula:

$$\xi_{Q_2} = \frac{\partial Q_2}{\partial I} \cdot \frac{I}{Q_2} = 0$$

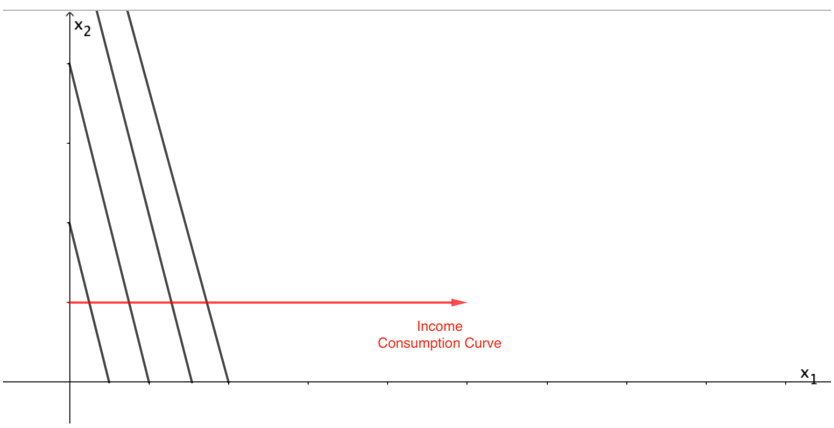
The income-elasticity is 0 because $Q_2 = \frac{1}{3}\frac{p_1}{p_2}$ does not depend on I.

5. Let's first calculate the income elasticity for a track day with motorbike, x_1

$$\xi = \frac{\partial x_1}{\partial I}\frac{I}{x_1} = \frac{1}{p_1}\frac{I}{\frac{I}{p_1}-\frac{1}{3}} = \frac{3I}{3I-p_1}$$

We only have to consider interior solutions. Therefore, we know that $3I > p_1$. So, we know that our denominator is positive. Besides, we also know, that the income elasticity for a track day with motorbike has to be larger than 1 because $3I > 3I - p_1$. The reason why the elasticity is more than 1 has to be searched in the fact that an increase in income does not affect the quantity demanded of good x_2. Therefore the quantity demanded of good x_1 is more than proportional to the increase in income.

6. Since the demand for good x_2 does not depend on income, the income-consumption curve is a horizontal line:



7. Suppose the price of x_1 increases. What is the impact on the consumption of both goods? Explain your answer.

We just have to look at the demand functions we derived in number two and so we can make the following statement:
If the price of x_1 increases the quantity demanded of good x_2 increases, while the quantity demanded of good x_1 decreases.

8. The impact of the change in the price of good x_1 on the demand of good x_2 can be decomposed on an income effect $= 0$ due to the fact that income elasticity for good $x_2 = 0$. Therefore the Substitution Effect equals the Total Effect. The increase in the quantity demanded it is then all explainable through the substitution effect.

Problem 6

1. In the equilibrium, the Q_P^S must be equal to Q_P^D. So, we can first find the equilibrium price as follow:

$$Q_P^S = 20P_P - 3P_F = 25 - 10P_P + 5P_C = Q_P^D$$

$$20P_P - 15 = 75 - 10P_P$$

$$P_P = 3 = P_P^*$$

Now, we can just fill in our P_P^* in either the supply or demand function to get the quantity in the equilibrium:

$$75 - 10 \cdot 3 = 45 = Q_P^*$$

Therefore, our equilibrium E^* is $(45, 3)$.

2. The consumer surplus is the monetary difference between the maximum amount that a consumer is willing to pay for the quantity of the good purchased and what the good actually costs. Because we have a linear demand curve, the consumer surplus is the triangle below the demand curve and above the equilibrium price. To calculate the consumer surplus, we first have to find the maximum price that consumers are willing to pay for Poporaya's Sushi. So, we have to find the intersection of the demand curve and the y-axis. Therefore, we can take our demand function $Q_P^D = 75 - 10P_P$ and set it equal to 0.

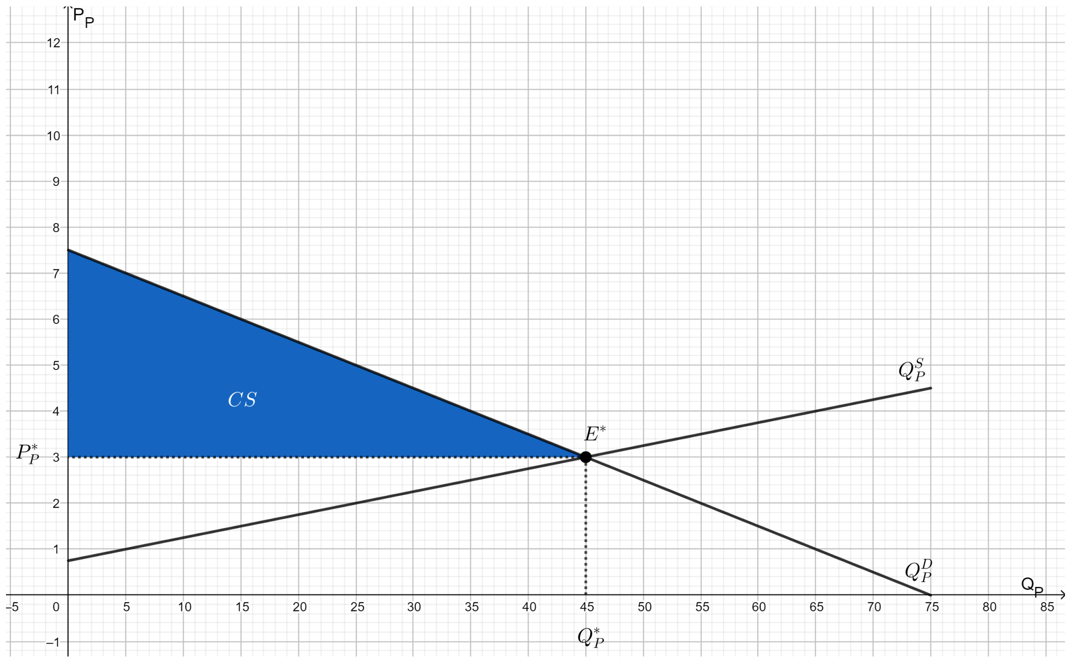
$$75 - 10P_P = 0$$
$$7.5 = P_P$$

Now we can calculate the triangle under the demand curve and above the equilibrium price as follows:

$$\tfrac{(7.5-3)\cdot 45}{2} = 101.25$$

Therefore, the consumer surplus is 101.25.

3. The following graph shows the consumer surplus, CS:



4. The price elasticity of the demand function can be calculated with the following formula:

$$\frac{\partial Q_P^D}{\partial P_P} \cdot \frac{P_P}{Q_P^D} = -10 \cdot \frac{3}{45} = -\frac{2}{3}$$

Therefore, the price elasticity of the demand function at the equilibrium is $-\frac{2}{3}$.

5. Let's calculate the cross-price elasticity of the demand for Poporoya's sushi with respect to the price of sushi sold by the average sushi competitor,PC, as follows:

$$\frac{\partial Q_P^D}{\partial P_C} \cdot \frac{P_C}{Q_P^D} = 5 \cdot \frac{10}{45} = -\frac{10}{9}$$

Therefore, the cross-price elasticity is $-\frac{10}{9}$.

6. The cross-price elasticity is positive. Therefore, the two goods are substitutes. When the price of sushi sold by the average sushi competitor, PC, increases, then the demand for Poporoya's sushi, Q_P^D, will increase as well.

7. We can make the same calculations like in number one, but we have to change P_F in our equation:

$$Q_P^{S'} = 20P_P - 3P_F = 25 - 10P_P + 5P_C = Q_P^D$$

$$20P_P - 21 = 75 - 10P_P$$

$$P_P = 3.2 = P_P^{**}$$

Now, we can just fill in our P_P^{**} in either the supply or demand function to get the quantity in the equilibrium:

$$75 - 10 \cdot 3.2 = 43 = Q_P^{**}$$

Therefore, our new equilibrium E^{**} is $(43, 3.2)$.

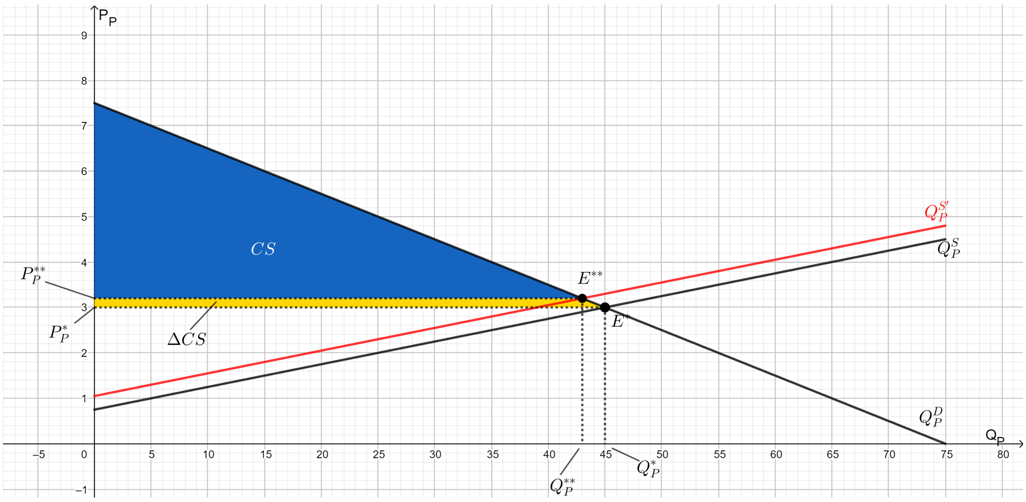
The increase in P_F shifts the supply curve upwards. This causes a higher price and a lower quantity in the our equilibrium.

8. The demand function did not change. Therefore, the the maximum price that consumers are willing to pay for Poporaya's Sushi is still 7.5. Now we can make the same calculation as in number 2 with the new equilibrium price and quantity:

$$\frac{(7.5-3.2)\cdot 43}{2} = 92.45$$

So, the new consumer surplus is 92.45. Therefore, we realize that the consumer surplus decreased by $101.25 - 92.45 = 8.8$. Therefore, $\Delta CS = -8.8$.

9. The yellow area in the following graph represents the change in consumer surplus, ΔCS (the red line represents the new supply curve $Q_P^{S'}$):



10. Little remark: in the following numbers, the results are always rounded to two decimal places.
The subsidy causes a change in the supply function. For each sushi sold at Poporoya, the owner now receives $P_P + 0.5$. Therefore, the new supply function $Q_P^{S''}$ is $= 20(P_P + 0.5) - 3P_F$. Now, we can again use the same method like in number 1 and 7, to derive the new market equilibrium:

$$Q_P^{S''} = 20(P_P + 0.5) - 3P_F = 25 - 10P_P + 5P_C = Q_P^D$$

$$20P_P - 5 = 75 - 10P_P$$

$$P_P = 2.67 = P_P^{***}$$

Now, we can just fill in our P_P^{***} in either the supply or demand function to get the quantity in the new equilibrium:

$$75 - 10 \cdot 2.67 = 48.33 = Q_P^{***}$$

Therefore, our equilibrium E^{***} is $(48.33, 2.67)$. With the subsidy, the supply curve shifts downward (the owner of Poporoya is willing to sell more Sushi at the original price).By comparing the initial equilibrium E^* with the new equilibrium E^{***}, we recognize, that the equilibrium quantity increased by 3.3 ($Q_P^{***} > Q_P^*$ and that the equilibrium price decreased by 0.33 ($P_P^{***} < P_P^*$).

11. Let's calculate first the new consumer surplus:

$$\frac{(7.5-2.67)\cdot 48.33}{2} = 116.72$$

Now we can calculate the difference between this new consumer surplus and the initial one to get the change in consumer surplus, ΔCS:

$$\Delta CS = 116.72 - 101.25 = 15.47$$

12. The government has to pay 0.5 per Sushi sold. Therefore the government has to pay $0.5 \cdot 48.33 = 24.17$. Therefore, the government's loss is 24.17.

Problem 7

1. Let's first calculate the marginal utility for U^L:

$$\begin{aligned} MU^L_{x_1} &= \frac{\partial U^L}{\partial x_1} = 2x_1 \\ MU^L_{x_2} &= \frac{\partial U^L}{\partial x_2} = 2x_2 \end{aligned}$$

Now we calculate the marginal utility for U^O:

$$\begin{aligned} MU^O_{x_1} &= \frac{\partial U^O}{\partial x_1} = 30x_1 \\ MU^O_{x_2} &= \frac{\partial U^O}{\partial x_2} = 2x_2 \end{aligned}$$

2. First, let's again calculate the MRS for U^L:

$$|MRS|^{U^L} = \frac{MU^L_{x_1}}{MU^L_{x_2}} = \frac{x_1}{x_2}$$

Levon is willing to give up $\frac{x_1}{x_2}$ Units of x_2 to get one more unit of x_1 Now, we also calculate the MRS for U^O:

$$|MRS|^{U^O} = \frac{MU^O_{x_1}}{MU^O_{x_2}} = \frac{15x_1}{x_2}$$

Ofelia is willing to give up $\frac{15x_1}{x_2}$ Units of x_2 to get one additional unit of x_1.

3. There are several ways to find out whether the MRS is increasing or decreasing. One possibility is to find the derivative of $|MRS|^{U^L}$ and $|MRS|^{U^O}$:

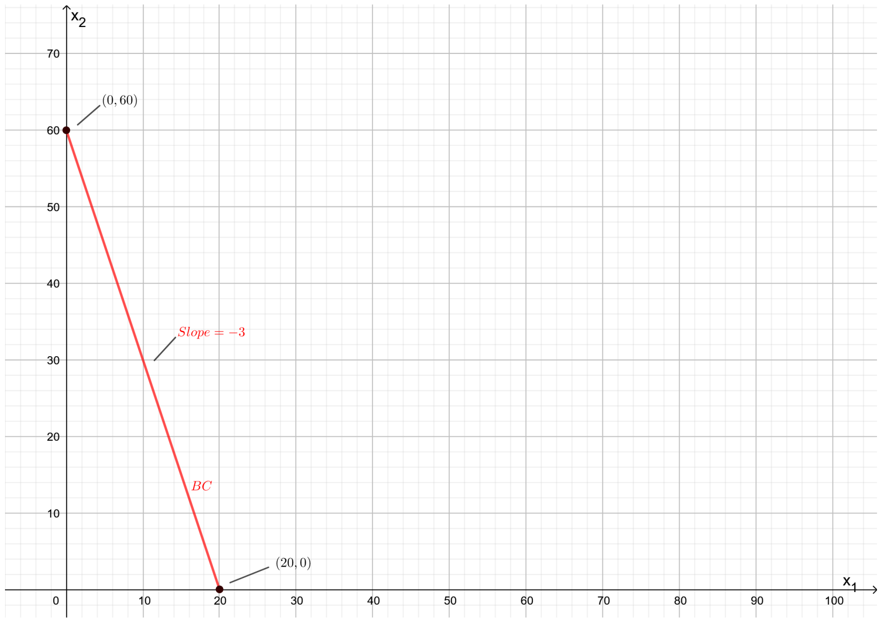
$$\frac{\partial |MRS|^{U^L}}{\partial x_1} = \frac{1}{x_2}$$

$$\frac{\partial |MRS|^{U^O}}{\partial x_1} = \frac{15}{x_2}$$

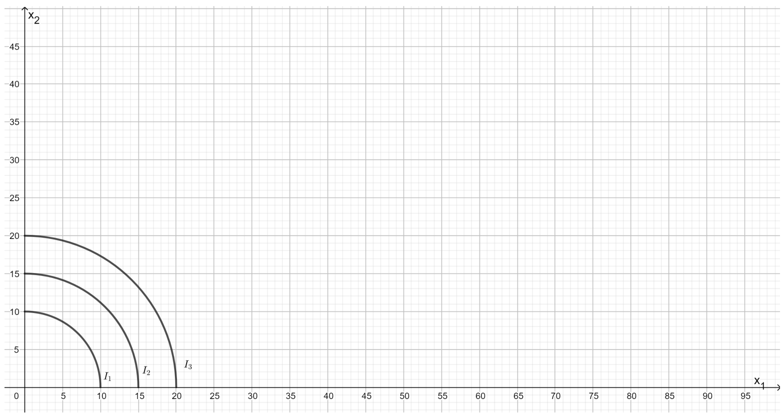
We see, that both expressions are positive. Therefore, we can say, that $|MRS|^{U^L}$ and $|MRS|^{U^O}$ are increasing in x_1.

We come to the same conclusion, when we just analyze both MRS. We can see that both, $|MRS|^{U^O}$. $|MRS|^{U^L} = \frac{x_1}{x_2}$ as well as $|MRS|^{U^O} = \frac{15x_1}{x_2}$, will increase, when x_1 increases. The economic interpretation is the following: When Levon and Ofelia start consuming one good, they become addicted. The more cups of ice cream, x_1 they already consume, the higher is the number of cups of coffee, x_2, they are willing to give up to get one more cup of ice cream, x_1.

4. The following figure shows the budget constraint for Levon:



5. The following figure shows 3 indifferent curves for Levon:



6. The figure in number 5 shows, that we have concave indifference curves. Wit our usual approach (using the tangency conditions and the budget constraint) we would get a minimum as a result. In case of concave indifference curves, the consumer only buys one good. Therefore, we can calculate the utilities of the corner bundles and compare the two results:

$$U(0,60) = 60^2 = 3600$$
$$U(20,0) = 20^2 = 400$$

Therefore, the optimal choice x^* is $(0,60)$.

An alternative way to find the optimal choice is the "highest indifference curve rule". The optimal bundle lies on the highest indifference curve that touches the budget line. The figure in number 7 will show - in accordance with our calculation above - that the point, where the highest indifference curve touches the budget line, is x^* is $(0,60)$.

7. The following figure shows the optimal bundle x^* (the point, where the highest indifference curve touches the budget line):

8. We have still concave indifference curves. Therefore, we can again compare the utilities in the two corners (we have to consider the fact that Levon can only buy 30 coffees after the increase in p_2).

$$U(0,30) = 30^2 = 900$$
$$U(20,0) = 20^2 = 400$$

Therefore, the optimal choice x^{**} is $(0,30)$.

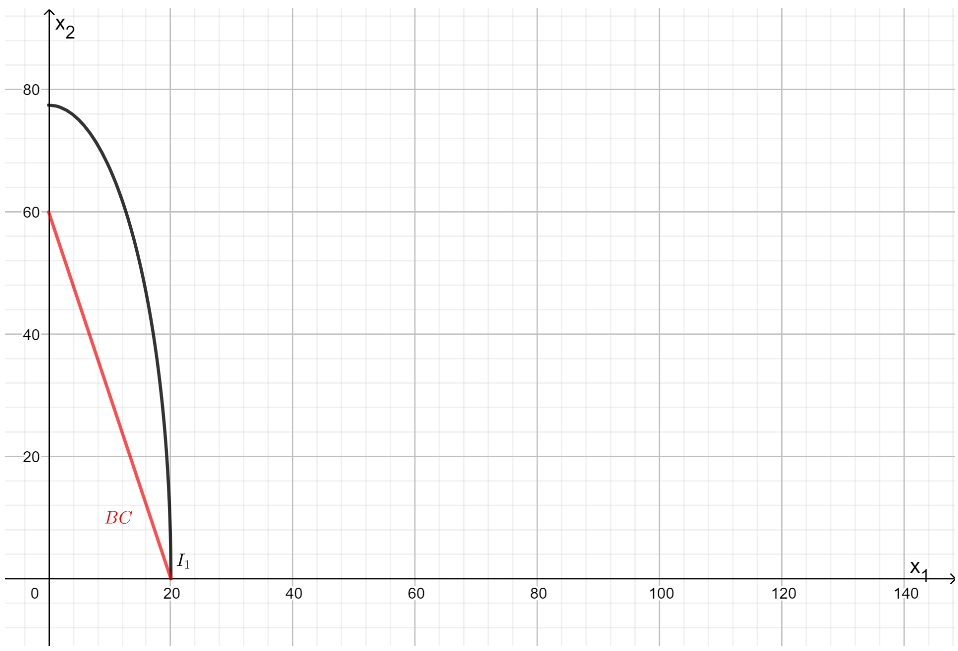
That means, that Levon still only consumes coffee. But, he can only afford to drink 30 cups of coffee after the increase in p_2.

9. In number 3, we already showed that the MRS of Ofelia's utility function is increasing in x_1. So, her indifference curves are also concave. Therefore, we can again just compare the utilities in the corners:

$$U(0, 60) = 60^2 = 3600$$
$$U(20, 0) = 15 * 20^2 = 6000$$

Therefore, the optimal choice for Ofelia, let's call it x^{***}, is $(20, 0)$.

Ofelia does not choose the same bundle as Levon. The reason is, that she doesn't have the same utility function. Therefore, her indifference curves are also different. In fact, the indifference curves look the following:



10. We already know that Ofelia will spend her whole income either on ice cream or coffee. She will spend all her income in cups of coffee, when she gets the higher utility by only consuming coffee compared to only consuming ice cream. Formally, we can say that she will only buy coffee when:

$$U(\tfrac{I}{p_1}, 0) < U(0, \tfrac{I}{p_2})$$

$$15(\tfrac{I}{p_1})^2 < (\tfrac{I}{1})^2$$

$$\sqrt{15} < p_1$$

When $p_1 > \sqrt{15}$, Ofelia will only consume coffee. The reason is, that the budget line become steeper and steeper when p_1 increases. When p_1 becomes too high, the point, where the highest indifference curve touches the budget line, will lie on the y-axis (this will be the case as soon as $\sqrt{15} < p_1$). The following figure shows the change of the budget constraint when p_1 changes. The red budget line shows the initial situation. The blue budget line represents the situation when $p_2 = 6$. The figure clearly shows that in the situation with the blue budget line, the point (0,60) lies on a higher indifference curve, than the point (10,0).