Microeconomics
Problem Set 3

Bocconi University Group number 30

due: October 9, 2019 at 8.00 pm

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Problem 1

Emily Brönte, whom you may know as the author of *Wuthering Heights*, can choose between two goods: writing poetry (interpret it as leisure, l) and eating apple pudding prepared by her sister Charlotte (interpret it as consumption, c), whose price p can be normalized to 1, i.e. $p = 1$. Her preferences can be represented by the following utility function:

$$U(l, c) = l^2 c$$

Her daily time endowment is $T = 18$ hours (she needs enough sleep, otherwise she can't be the brilliant author of Wuthering Heights!).

Emily has recently got a teaching position at Roe Head school. Let h be the number of hours per day that Emily devotes to teaching at a pre-tax hourly wage $w = £10$ (pounds). Currently, Emily has no other income apart from the one she earns as a teacher.

1. Suppose that the local government imposes a labor tax τ on the hourly wage. Thus, the after-tax wage is $(1-\tau)w$. Derive analytically Emily's budget constraint, calculate the intercepts and the slope.

   The general formula for the budget constraint is:

   $$wl + pc = wT + \bar{I}$$

   $$pc = wT + \bar{I} - wl$$
   $$c = (\tfrac{wT}{p} + \tfrac{\bar{I}}{p}) - \tfrac{w}{p}l$$

   This expression shows the vertical intercept $= (\frac{wT}{p} + \frac{\bar{I}}{p})$ and the slope $= -\frac{w}{p}$. To find Emily's budget constraint, the intercepts and the slope, we have to consider that $\bar{I} = 0$, $p = 1$ and $w = 10$. Besides, there is the labor tax τ. Therefore, the budget constraint looks as follows:

   $$c = (1-\tau)wT - (1-\tau)wl$$

   $$c = (1-\tau)180 - (1-\tau)10l$$

   With all these information, we can now also calculate the intercepts and the slope:

   Vertical intercept $= (1-\tau)wT = (1-\tau)180$
   Horizontal intercept $= T = 18$
   Slope = -$(1-\tau)w = -(1-\tau)10$

2. Provide an economic interpretation for the intercepts and the slope of the budget constraint.

   Vertical intercept:
   The maximal amount that Emily can spend for consumption when she decides to have no leisure and to work for $T = 18$ hours.

   Horizontal intercept:
   The horizontal intercept shows the maximal amount of hours, Emily can spend on leisure. It is given by the daily time endowment.

Slope:
The slope shows the relative price of writing poetry. It shows how many apple puddings Emily has to give up for one additional unit of leisure (writing poetry).

3. Derive analytically the demand for leisure, $l(w,\tau,T)$, and the demand for consumption, $c(w,\tau,T)$ (hint: solve for the optimal bundle as a function of w, τ and T).
We have to consider the following equations to find the demand for leisure

$$\begin{cases} |MRS| = |MRT| \\ c = (1-\tau)wT - (1-\tau)wl \\ T = h + l \end{cases}$$

$$\begin{cases} \frac{MU_l}{MU_c} = (1-\tau)w \\ c = (1-\tau)wT - (1-\tau)wl \\ T = h + l \end{cases}$$

$$\begin{cases} \frac{2c}{l} = (1-\tau)w \\ c = (1-\tau)wT - (1-\tau)wl \\ T = h + l \end{cases}$$

$$\begin{cases} c = \frac{l(1-\tau)w}{2} \\ c = (1-\tau)wT - (1-\tau)wl \\ T = h + l \end{cases}$$

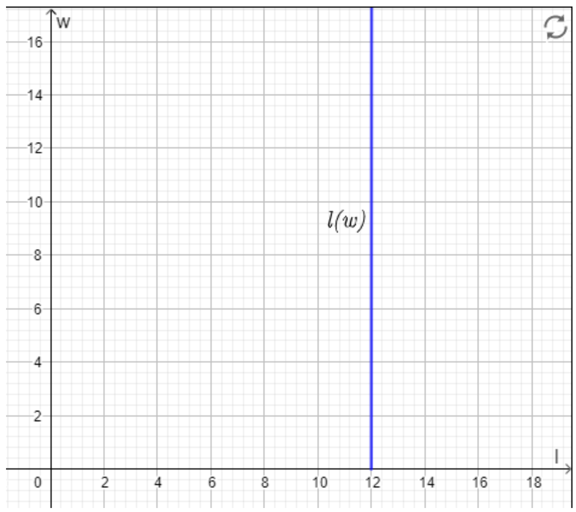
By solving these equations for l and c respectively, we get:

$$l(w,\tau,T) = \frac{2(1-\tau)wT}{3(1-\tau)w} = \frac{2T}{3}$$

$$c(w,\tau,T) = \frac{(1-\tau)wT}{3}$$

4. On a properly labeled graph, draw the demand for leisure $l(w)$.

   We can fill $T = 18$ in our demand function, we derived above and we get $l(w) = 12$:



5. What can we argue about the slope of the demand for leisure? What is the economic interpretation of this slope?

   The graph in number 4 shows, that the demand for leisure $l(w)$ is a vertical line. The reason is, that in case of Emily, leisure does not depend on the wage. Emily always wants to spend $\frac{2}{3}T = 12$ hours writing poetry. So, since the demand for leisure is vertical, the slope can not be defined.

6. Define the labor supply curve $h(w)$.

   The labor supply curve $h(w)$ shows the hours an individual is willing to work as an function of the wage, w.

7. Derive analytically Emily's labor supply $h(w)$.

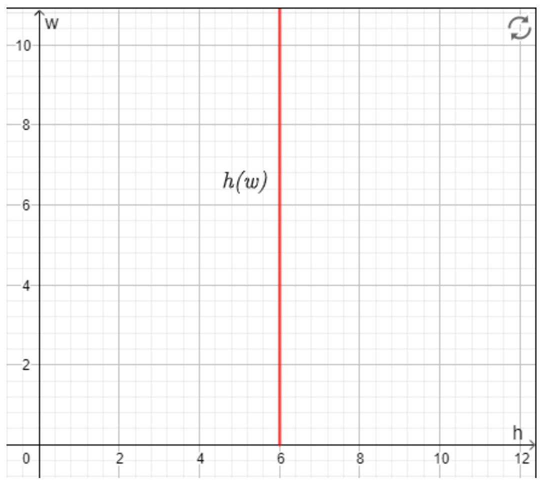
   By subtracting Emily's demand for hours of leisure at each wage from T, we obtain her labor supply curve:

$$T = l + h$$
$$T = \frac{2T}{3} + h$$
$$h = \frac{1T}{3}$$

By substituting $T = 18$ into our equation above, we get:

$$h(w) = 6$$

8. On a properly labeled graph, draw the labor supply.



9. What can we argue about the slope of the labor supply? What is the economic interpretation of this slope?
The labor supply $h(w)$ is also a vertical line. The reason can be found in the vertical demand for leisure $l(w)$. This means, that Emily always wants to work for 6 hours (no matter how much she can earn per working hour). Therefore, the slope of the labor supply is again undefined.

10. Suppose now that the tax is $\tau = 0.4$. Using the information on T, w, p given above, calculate Emily's optimal bundle $E^* = (l^*, c^*)$.
We can just substitute $\tau = 0.4$ into the demand functions we derived in number 3, fill in $T = 18$, $w = 10$ and we get:

$$l(w, \tau, T) = \frac{2T}{3} = 12$$

$$c(w, \tau, T) = \frac{(1-0.4)wT}{3} = 36$$

Therefore, the optimal bundle $E^* = (l^*, c^*)$ is:

$$E^* = (12, 36)$$

11. How many hours h^* will Emily spend teaching at Roe Head school if she chooses the optimal bundle (l^*, c^*)?

$$\begin{aligned} T &= l + h \\ 18 &= 12 + h \\ h &= 6 \end{aligned}$$

Therefore, $h^* = 6$.

12. With no surprise, Roe Head school has been able to enroll many more students since Emily joined as a teacher (brava!!!). To provide an incentive for Emily to spend more hours at teaching classes, Roe Head school asks the local government to eliminate the labor tax (that is, to set $\tau = 0$).

Derive analytically the demand for leisure, $l(w, \tau, T)$, and the demand for consumption, $c(w, \tau, T)$, when the tax is $\tau = 0$. (hint: solve for the optimal bundle as a function of w and T; τ is now set to 0).
Similar to number 3, we have to consider the following equations to find the demand for leisure

$$\begin{cases} |MRS| = |MRT| \\ c = wT - wl \\ T = h + l \end{cases}$$

$$\begin{cases} \frac{MU_l}{MU_c} = w \\ c = wT - wl \\ T = h + l \end{cases}$$

$$\begin{cases} \frac{2c}{l} = w \\ c = wT - wl \\ T = h + l \end{cases}$$

$$\begin{cases} c = \frac{lw}{2} \\ c = wT - wl \\ T = h + l \end{cases}$$

By solving these equations for l and c respectively, we get:

$$l(w, \tau, T) = \frac{2wT}{3w} = \frac{2T}{3}$$

$$c(w, \tau, T) = \frac{wT}{3}$$

13. Derive analytically the labor supply $h(w)$ without the labor tax.

$$T = l + h$$
$$T = \frac{2T}{3} + h$$
$$h = \frac{1T}{3}$$

14. Find analytically the new optimal bundle $E^{**} = (l^{**}, c^{**})$ that maximizes Emily's utility without the labor tax.

    Let's just substitute $T = 18$ and $w = 10$ into the demand functions we derived in number 12:

$$l(w, \tau, T) = \frac{2wT}{3w} = \frac{2T}{3} = 12$$

$$c(w, \tau, T) = \frac{wT}{3} = 60$$

    Therefore, the optimal bundle $E^{**} = (l^{**}, c^{**})$ is:

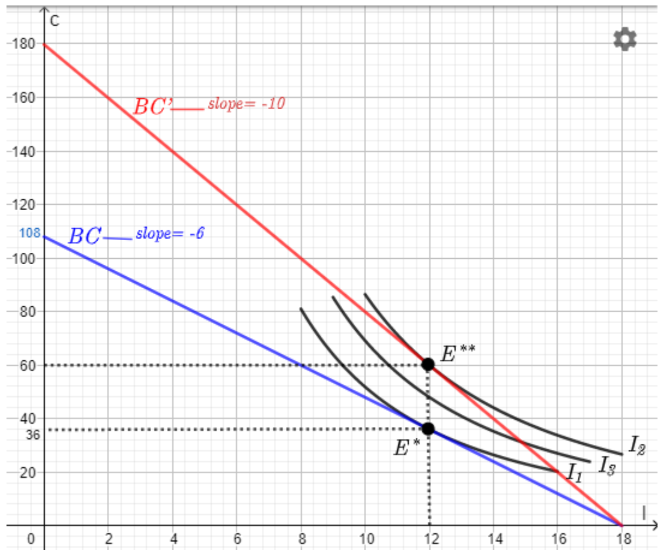
$$E^{**} = (12, 60)$$

15. Calculate the optimal number of hours h^{**} that Emily will supply when there is no wage tax.

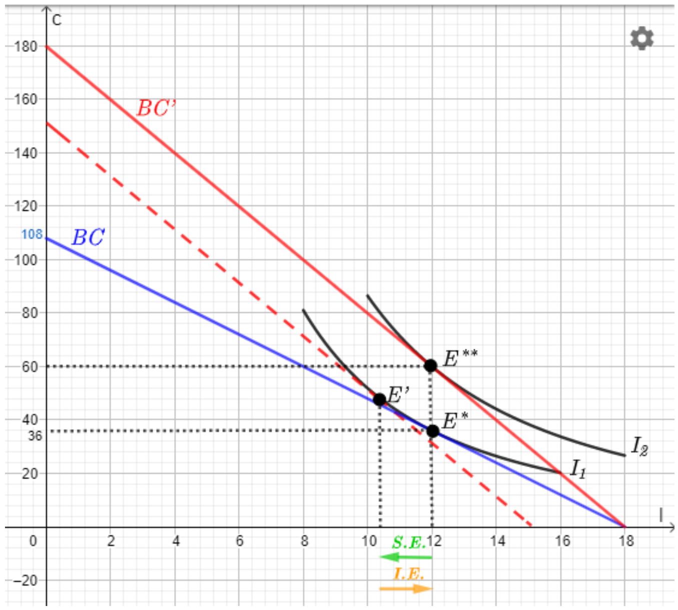
    In number 13, we derived $h = \frac{1T}{3}$. Therefore:

$$h^{**} = 6.$$

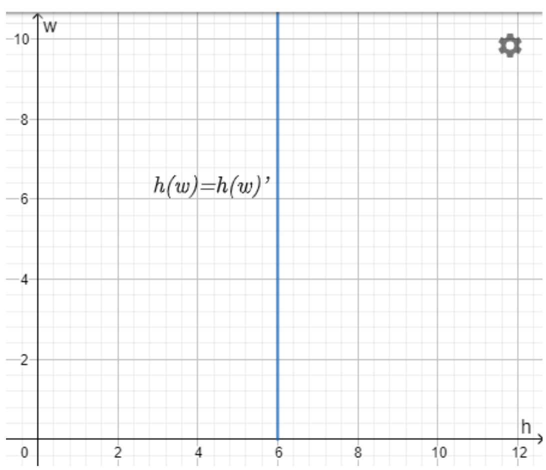
16. Using a graph in which you label the $x-$axis with l and the $y-$axis with c, draw Emily's budget constraint (show the intercepts and the slope), three representative indifference curves, and the optimal bundles E^* and E^{**}.
The wage of Emily will be higher, when the labor tax is eliminated. This leads to the new budget constraint BC'.



17. On a properly labeled graph, show the substitution and income effects **only** with respect to Emily's choice of writing poetry, l, (no calculations required!). The wage of Emily will be higher, when the labor tax is eliminated. This leads to the new budget constraint BC'. This means, that writing poetry becomes relatively more expensive. Therefore, the substitution effect (the movement from E^* to E'), in this scenario is negative. But, we also have to consider the income effect. As her wage rises, if Emily works the same number of hours as before, she will have a higher income. The income effect is represented in the graph by the movement from E' to E^{**}. The figure shows that the income effect is positive. This means, that Emily consumes more leisure when her income increases and therefore, writing poetry is a normal good. Since $l^* = l^{**}$, the income effect is equal to the substitution effect.



18. On a properly labeled graph, draw Emily's labor supply $h(w)$ with and without the tax.
    Like in number 7 and 13 calculated, the labor supply with the tax, $h(w)$, and the labor supply without the tax, $h(w)'$ are both times the same:



19. Will Roe Head school be able to reach its goal of making Emily work more through this tax policy enacted by the local government?
    No, the explanations above show, that she will work in both scenarios 6 hours.

20. Suppose now that the local government decides to reject Roe Head school proposal; therefore, the tax of $\tau = 0.4$ on the hourly wage is still in place. At the same time, the price of the apple pudding has now increased to $p' = 3$. Write Emily's new budget constraint.

$$(1-\tau)wl + pc = (1-\tau)wT$$
$$6l + 3c = 108$$

21. Derive analytically the optimal bundle $E^{***} = (l^{***}, c^{***})$ that maximizes Emily's utility.

Similar to number 3 and 12, we have to consider the following equations to find the new optimal bundle:

$$\begin{cases} |MRS| = |MRT| \\ 6l + 3c = 108 \\ T = h + l \end{cases}$$

$$\begin{cases} \frac{MU_l}{MU_c} = \frac{(1-\tau)w}{p} \\ 6l + 3c = 108 \\ T = h + l \end{cases}$$

$$\begin{cases} \frac{2c}{l} = \frac{6}{3} \\ 6l + 3c = 108 \\ T = h + l \end{cases}$$

$$\begin{cases} c = l \\ 6l + 3c = 108 \\ T = h + l \end{cases}$$

By solving these equations for l and c respectively, we get the optimal bundle $E^{***} = (l^{***}, c^{***})$:

$$E^{***} = (12, 12)$$

22. How many hours h^{***} will Emily spend teaching at Roe Head school?

$$T = l + h$$
$$T = 12 + h$$
$$h = 6$$

Therefore: $h^{***} = 6$

23. Compare h^{*}, h^{**}, h^{***} and discuss your findings.
All our calculations showed that:

$$h^{*} = h^{**} = h^{***}.$$

The reason is, that writing poetry, l, does not depend on the wage, w and the price, p. Therefore, also the hours Emily is teaching per day, h, will remain the same after a change in wage or price. Let's show, that this statement is true:

$$\begin{cases} |MRS| = |MRT| \\ (1-\tau)wl + pc = (1-\tau)wT \\ T = h + l \end{cases}$$

$$\begin{cases} \frac{2c}{l} = \frac{(1-\tau)w}{p} \\ pc = (1-\tau)wT - (1-\tau)wl \\ T = h + l \end{cases}$$

$$\begin{cases} c = \frac{(1-\tau)wl}{2p} \\ c = \frac{(1-\tau)wT}{p} - \frac{(1-\tau)w}{p} l \\ T = h + l \end{cases}$$

When we solve these equations, we will always get $l = \frac{2T}{3}$ and $h = \frac{1T}{3}$, no matter what wage or price we have.

24. Aunt Elizabeth is worried about the increase in the price of Emily's favorite dessert, apple pudding. Thus, she is willing to give Emily (her favorite niece) some extra money $\bar{I} = 27$. Derive analytically Emily's new budget constraint, calculate the intercepts and the slope. Budget constraint:

$$(1-\tau)wl + pc = (1-\tau)wT + \bar{I}$$
$$6l + 3c = 135$$

Vertical intercept:

$$(\tfrac{(1-\tau)wT}{p} + \tfrac{\bar{I}}{p}) = 45$$

Horizontal intercept:

$$T = 18.$$

If we drew the budget constraint, we would realize that there is a kink caused bey the extra money of Aunt Elizabeth.

Slope:

$$-\frac{\tau)w}{p} = -2$$

25. Find analytically the optimal bundle $E^{****} = (l^{****}, c^{****})$ that maximizes Emily's utility.

$$\begin{cases} |MRS| = |MRT| \\ (1-\tau)wl + pc = (1-\tau)wT + \bar{I} \\ T = h + l \end{cases}$$

$$\begin{cases} \frac{2c}{l} = \frac{(1-\tau)w}{p} \\ pc = (1-\tau)wT + \bar{I} - (1-\tau)wl \\ T = h + l \end{cases}$$

$$\begin{cases} c = \frac{(1-\tau)wl}{2p} \\ c = \frac{(1-\tau)wT+\bar{I}}{p} - \frac{(1-\tau)w}{p}l \\ T = h + l \end{cases}$$

$$\begin{cases} c = l \\ c = 45 - 2l \\ T = h + l \end{cases}$$

By solving these equations for l and c respectively, we get the optimal $E^{****} = (l^{****}, c^{****})$:

$$E^{****} = (15, 15)$$

26. Will Emily work more or fewer hours h^{****} in the new optimal bundle? Explain your answer.

$$T = l + h$$
$$T = 15 + h$$
$$h = 3$$

Therefore: $h^{***} = 3$

So, Emily will work 3 hours less. The reason is, that leisure, l depends on the exogenous income, $\bar{I}$. The higher the exogenous income, the less she will work. We can proof this by deriving the new demand function of Emily. In number 25, we saw that:

$$\begin{cases} c = \frac{(1-\tau)wl}{2p} \\ c = \frac{(1-\tau)wT+\bar{I}}{p} - \frac{(1-\tau)w}{p}l \\ T = h + l \end{cases}$$

Therefore:

$$\frac{(1-\tau)wl}{2p} = \frac{(1-\tau)wT+\bar{I}}{p} - \frac{(1-\tau)w}{p}l$$
$$l(3(1-\tau)w)) = 2(1-\tau)wT + 2\bar{I}$$
$$l = \frac{2T}{3} + \frac{2}{3}\frac{\bar{I}}{(1-\tau)w}$$

27. Time has elapsed. Two things have happened in the meantime. First, the price of apple pudding has dropped to the initial level, that is, $p = 1$. Second, Emily has been finally able to publish her masterpiece, *Wuthering Heights*. Since then, her preferences have changed (uuhhhhhhh! this sounds quite interesting!!!! let's see

what it's going to happen... this Microeconomics stories are better than watching a movie!!!!) and can be described by the following utility function:

$$U(l, c) = min(2l, c)$$

The remaining variables are identical to the very beginning of this problem, that is, $T = 18, w = 10, \tau = 0.4,$

28. What type of goods are leisure and consumption for Emily? Explain your answer.

    The typical utility function for perfect complements is $U(x_1, x_2) = min(ix_1, jx_2)$. Therefore, the new utility function shows, that consumption and leisure are perfect complements. Emily wants to consume them only in fixed proportions.

29. Derive analytically Emily's new budget constraint, calculate the intercepts and the slope.

    Budget constraint:

    $$(1-\tau)wl + pc = (1-\tau)wT$$
    $$6l + c = 108$$

    Vertical intercept:

    $$\frac{(1-\tau)wT}{p} = 108$$

    Horizontal intercept:

    $$T = 18.$$

    Slope:

    $$-\frac{\tau)w}{p} = -6$$

30. Derive analytically the new demand for leisure, $l(w, \tau, T)$ and the new demand for consumption, $c(w, \tau, T)$.

$$\begin{cases} 2l = c \\ (1-\tau)wl + pc = (1-\tau)wT \end{cases}$$

$$\begin{cases} 2l = c \\ (1-\tau)wl + 2l = (1-\tau)wT \end{cases}$$

$$\begin{cases} 2l = c \\ (1-\tau)wl + 2l = (1-\tau)wT \end{cases}$$

When we solve for l and c respectively, we get:

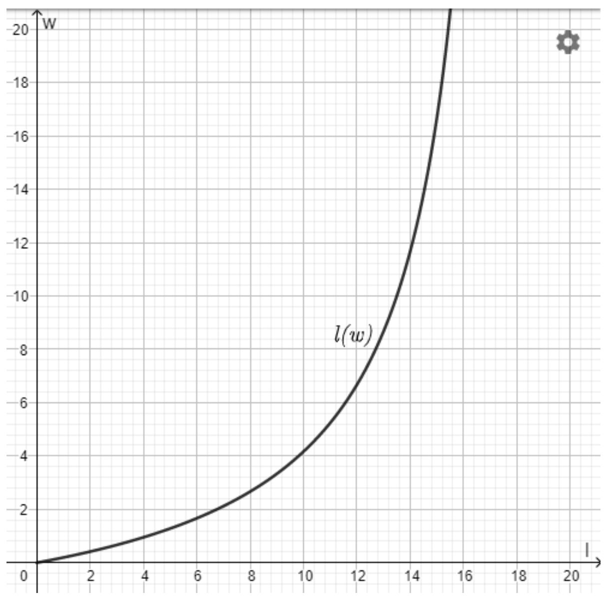
$$l(w, \tau, T) = \tfrac{(1-\tau)wT}{2+(1-\tau)w}$$

$$c(w, \tau, T) = 2\tfrac{(1-\tau)wT}{2+(1-\tau)w}$$

31. On a properly labeled graph, draw Emily's demand for leisure, $l(w)$.
    We have to rearrange the demand for leisure which we derived in number 30 and we get:

$$w = \frac{2l}{(1-\tau)T-(1-\tau)l} = \frac{2l}{10.8-0.6l}$$

Therefore, $l(w)$ looks as follows:



32. Derive analytically Emily's new labor supply, $h(w)$. Again, we want to derive the number of hours supplied, h, as a function of wage, w.In number 30 we found that $l(w,\tau,T) = \frac{(1-\tau)wT}{2+(1-\tau)w}$.Let's now substitute $T = 18$ and $(1-\tau) = 0.6$:

$$l(w,\tau,T) = \frac{10.8w}{2+0.6w}$$

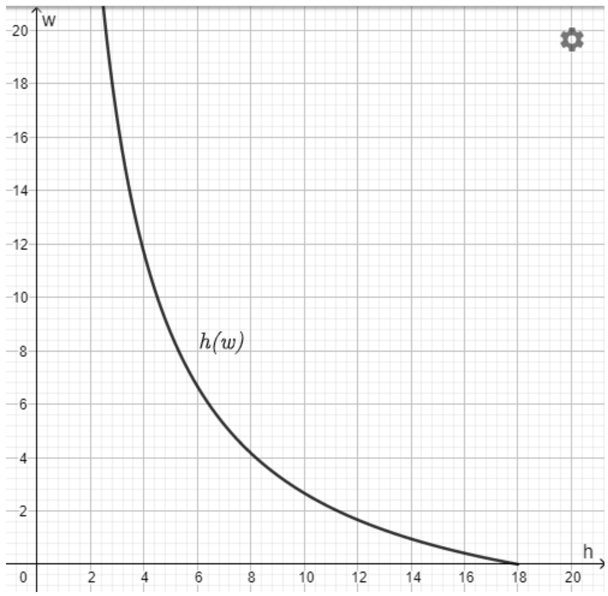
Further, we know that $T = 18 = h + l$. By substituting our derived l and solving for h we get:

$$h(w) = 18 - \frac{10.8w}{2+0.6w}$$

33. On a properly labeled graph, draw Emily's new labor supply, $h(w)$.
    Let's rearrange our solution in number 32:

$$w = -3.33 + \frac{60}{h}$$

The graph below shows this new labor supply function $h(w)$:



34. Compare the labor supply you derived in point (7) and the labor supply you found here. Are they similar or different? What is the economic interpretation of your findings?
The labor supply in point 7 was vertical. Therefore, it did not depend on the wage and Emily always worked the same number of hours, no matter how high her wage was. Our new labor supply does depend on the wage and it has a negative slope. This means, that Emily will work less the higher the wage becomes. Therefore, we can say that the income effect dominates the substitution effect.

35. Find analytically the optimal bundle $E^{*****} = (l^{*****}, c^{*****})$ that maximizes Emily's utility with the new preferences.

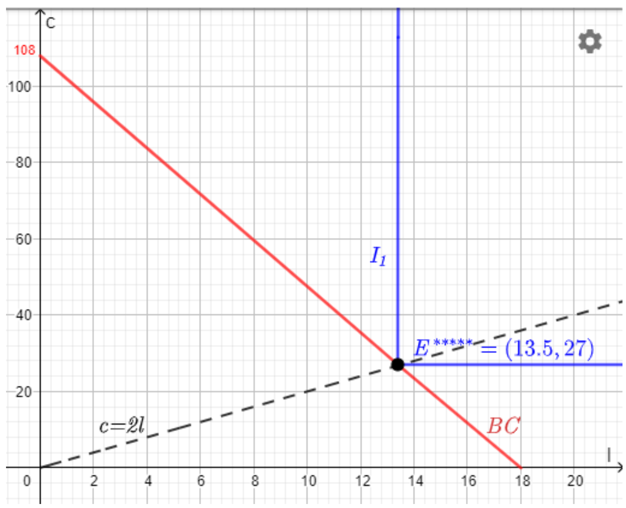
$$\begin{cases} 2l = c \\ \text{6l + c} = 108 \end{cases}$$

Therefore, we get for the new optimal bundle $E^{*****} = (l^{*****}, c^{*****})$:

$$E^{*****} = (13.5, 27)$$

Remark: we could also fill in all the numbers into our demand functions, we derived in number 30 and we would get the same results.

36. On a properly labeled graph, draw Emily's optimal bundle E^{*****}.



37. How many hours, h^{*****}, will Emily work at the new optimal bundle?

$$T = l + h \ 18 = 13.5 + h \ h = 4.5$$

Therefore, $h^{*****} = 4.5$. We would get the same result by just substituting all relevant variables into the equation we derived in number 32.

38. Will she work more, fewer, or the same number of hours in h^{*****} compared to the number of hours in h^*? What is the economic interpretation of this finding? h^* was 6. Therefore, Emily now works 1.5 hours less. In the first scenario, Emily always wanted to work the same amount of hours ($\frac{1}{3}T$). So, when the wage changed, the substitution and income effect worked in opposite directions and neither one nor the other was dominant. This can be seen in the graph in number 17. Now in the newest scenario, we have a different labor supply. In number 33

and 34, we showed that this labor supply was downward sloping. Emily wants to work less when her wage raises, because the income effect became dominant (little remark: we see that in both scenarios, the income effect is working in the opposite direction of the substitution effect. Therefore, we can say, that leisure is both times a normal good for Emily).

Problem 2

1. Harvey's intertemporal budget constraint is:

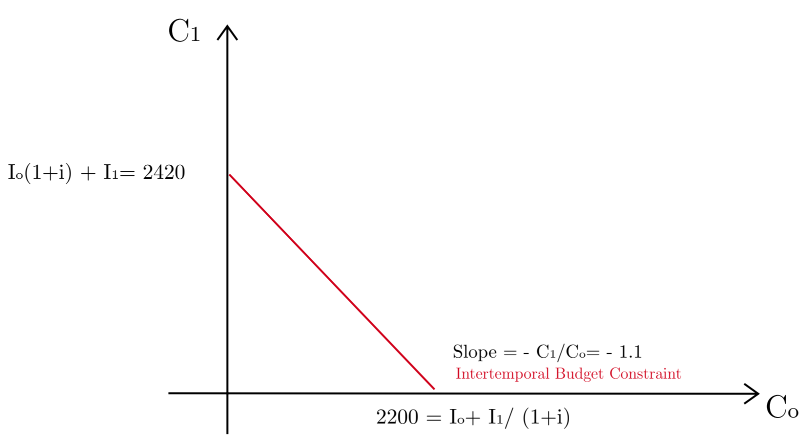
$$
\begin{aligned}
c_1 &= (I_0 - c_0)(1+i) + I_1 \\
c_1 &= I_0(1+i) - c_0(1+i) + I_1 \\
c_1 + c_0(1.1) &= 1000(1.1) + 1320 \\
c_1 + c_0(1.1) &= 2420 \ \$
\end{aligned}
$$

2. The intercepts and the slope of the intertemporal budget constraint are:

$$
\begin{aligned}
c_1 &= 2420 - 1.1c_0 \rightarrow \text{Vertical intercept: } 2420 \ \$ \\
c_0 &= 2200 - \tfrac{c_1}{1.1} \rightarrow \text{Horizontal intercept: } 2200 \ \$ \\
Slope &= -\tfrac{2420}{2200} = -1.1
\end{aligned}
$$

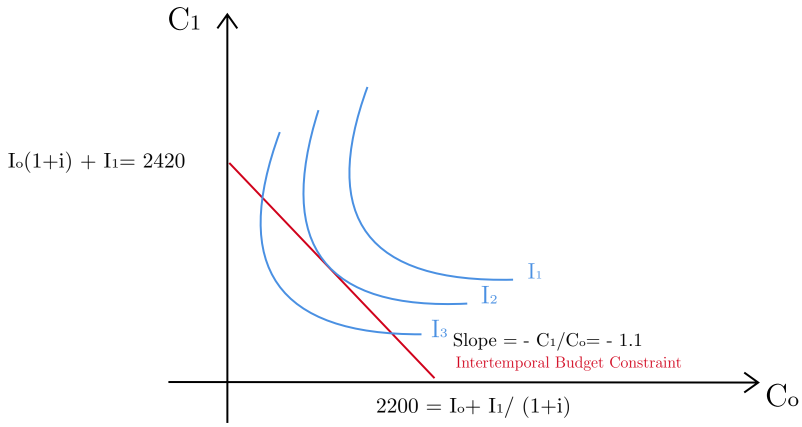
3. The vertical intercept shows is the amount of money that Harvey can spend for consumption next year, if he would save all of his income this year at an interest rate of 10 %. Therefore, Harvey would only consume in the second period.
The horizontal intercept is the amount of money that Harvey can spend for consumption this year, if he decides to spend also next year's income borrowed at the market rate. This means, that Harvey would only consume in the first period.
The slope explains the trade-off that Harvey has to face between current and future consumption. How much of C_1, Harvey has to give up in order to obtain an additional unit of C_0.

4.



5. The utility function that describes Harvey's preferences is a Cobb-Douglas, therefore the two goods are imperfect substitutes.

6.



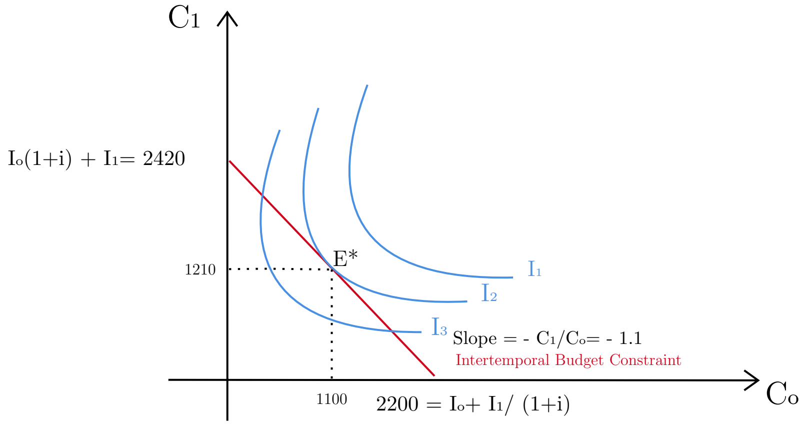
7. The optimal bundle E^* can be evaluated as we were used to do in the consumption choice through a system with the tangent condition ($|MRS| = \frac{MUc_o}{MUc_1} = 1.1 = |MRT| = |-\frac{c_1}{c_o}|$)

$$\begin{cases} |MRS| = |MRT| \\ c_1 + c_o(1.1) = 2420\$ \end{cases}$$

$$\begin{cases} c_1 = (1.1)c_o \\ 2c_o(1.1) = 2420\$ \end{cases}$$

$$\begin{cases} c_1 = 1210\$ \\ c_o = 1100\$ \end{cases}$$

8.



9. To know if Harvey will save or borrow at the optimal bundle E^*, we start by evaluating the endowment bundle which is:

$$(I_0, I_1) = (1000\ \$, 1320\ \$)$$

We can now conclude that, because Harvey is consuming this year 1100\$ while his $I_0 = 1000\$$ he can be classified as a borrower. Therefore the amount borrowed is: $I_0 - c_0 = 100\$$

10. Harvey Specter's new intertemporal budget constraint, due to a change in interest rate ($i' = 20\%$) can be evaluated as follows:

$$c_1 + c_0(1 + i') = I_0(1 + i') + I_1$$
$$c_1 + c_0(1.2) = I_0(1.2) + I_1$$
$$c_1 + c_0(1.2) = 1000(1.2) + 1320$$
$$c_1 + c_0(1.2) = 2520\ \$$$

$$\begin{cases} \text{Vertical intercept} = 2520\ \$ \\ \text{Horizontal intercept} = \frac{2520\ \$}{1.2} = 2100\ \$ \\ \text{Slope} = -\frac{c_0}{c_1} = -\frac{2520\ \$}{2100\ \$} = -1.2 \end{cases}$$

11. To derive Harvey's optimal bundle E^{**} we proceed as above:

$$\begin{cases} \frac{c_1}{c_0} = 1.2 \\ c_1 + c_0(1.2) = 2520 \ \$ \end{cases}$$

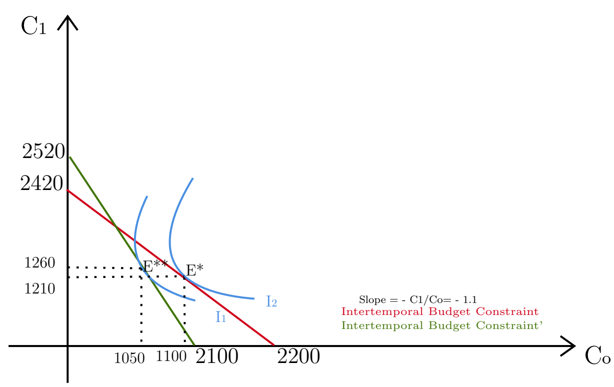
$$\begin{cases} c_1 = 1260 \ \$ \\ c_0 = 1050 \ \$ \end{cases}$$

12. From the comparison of this two optimal bundles:

$$E^{*} = (1100 \ \$, 1210 \ \$)$$
$$E^{**} = (1050 \ \$, 1260 \ \$)$$

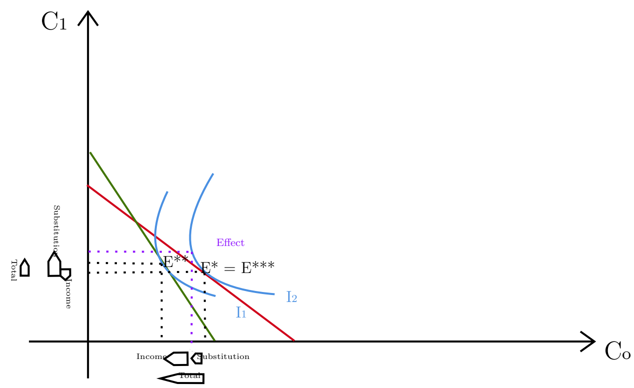
We can evaluate that both the behaviour lead to borrow money. We can also observe that after the increase of the interest rate, Harvey will consume less in the first period and more in the second period compared to the situation before. The reason is, that in the optimal bundle E^{**}, Harvey decides to borrow less money in the first period because borrowing became more costly.

13.



14. The increase in interest rates causes a movement from optimal bundle E^{*} to the new optimal bundle E^{**}. This movement is the total effect. The total effect consists of the substitution and income effect. When the interest rate increases, it's more convenient for the individual to save money because borrowing becomes more expensive. Therefore the substitution effect encourages to exchange borrowings in favour of savings. This means, that due to the substitution effect, Harvey

will borrow less in the first period. Like our graph shows, this effect causes a decrease in c_0 and an increase in c_1. Our graph shows also, that in the first period, the income effect works in the same direction like the substitution effect. The reason is, that Harvey is a borrower. Therefore, an increase in the interest rate makes him relatively poorer. Therefore, the income effect causes a decrease in consumption in both periods.



15. In the case that the interest rate for savings (I_S) is different from the interest rate for borrowings (I_B), the intertemporal budget constraint will observe a different slope between the part on the right of the endownment bundle and on the left.

    In our case, Harvey could be a saver if his $c_0 \in [0\$, 1000\$]$. In this case, the interest rate for savings would be relevant:

$$c_1 = (I_0 - c_0)(1+i) + I_1$$
$$c_1 = (1000 - c_0)(1.15) + 1320$$
$$c_1 + c_0(1.15) = 2470$$

$$\begin{cases} c_1 = 2470 \ \$ \\ c_0 = 2148 \ \$ \end{cases}$$

    We can observe that c_0 violates the conditions of existence and it can't be true!

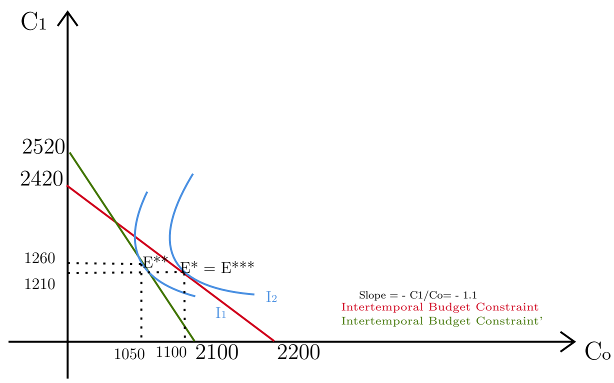
    We can conclude that the only condition valid is for the interest rate of 10%. Therefore, the budget constraint is:

$$c_1 + c_0(1.10) = 2420\$ \text{ for } c_0 \in [1000\$, 2420\$]$$

    The intercepts and the slope are:

$$c_1 = 2420 - 1.1c_0 \rightarrow \text{Vertical intercept: } 2420 \text{ \$}$$
$$c_0 = 2200 - \frac{c_1}{1.1} \rightarrow \text{Horizontal intercept: } 2200 \text{ \$}$$
$$Slope = -\frac{2420}{2200} = -1.1$$

16. The red budget constraint in the following graph represents the budget constraint for the very first scenario and also the budget constraint for the scenario in number 15. The graph also shows the slope, which is -1.1.



17. In number 15, we showed that the new budget constraint is equal to the budhet constraint in number 1. Therefore, the optimal bundle $E^{***} = E^{*}$, and $E^{***} = (1100 \text{ \$}, 1210 \text{ \$})$

18. The optimal bundle E^{***} it's already included in the graph at point 16, since it is equal to E^{*}.

19. E^{*}, E^{**} and E^{***} are all bundles for which Harvey results in being a borrower because he consumes always more (this year) than his actual income is.

$$c_0 > I_0$$

We also saw that $c_0^{**} < c_0^{*} = c_0^{***}$ and $c_1^{**} > c_1^{*} = c_1^{***}$. This was caused by the higher interest rate that made borrowing more expensive. Besides, we see, that E^{**} is on a lower indifference curve than $E^{*} = E^{***}$. Therefore, we can conclude that his utility in $E^{*} = E^{***}$ is bigger than the on in E^{**}.

20. We can derive Harvey's saving function $S(i)$ as follows:

$$S(i) = I_0 - c_0$$
$$S(i) = I_0 - (I_0 + \tfrac{I_1 - c_1}{1+i})$$
$$S(i) = \tfrac{I_1 - c_1}{1+i}$$

Problem 3

Bill Gates is a very young and creative guy, always smiling (his nickname is indeed "Happy Boy"!). Bill is working with his friend Paul and they have just founded their start up, called Micro-soft (a combination of the words "microcomputer" and "software").

In the current period, t_0, Bill's income is relatively small, $I_0 = 20$, but he is confident that in the future, t_1, he will become very rich, $I_1 = 100$.

Let c_0 and c_1 be the consumption bundles Bill chooses in the current and in the future periods, respectively. Prices of consumption goods are normalized to 1.

Let S_0 be Bill's savings in the current period in case he decides to save (and deposit his savings in a bank). Let B_0 be Bill's borrowing this year if he decides to borrow money (e.g., from a bank).

The market nominal interest rate $i = 25\%$ (uuhhh la Peppa!!!!!!!!! is this Disneyland????) for both borrowers and lenders.

1. Write analytically Bill's intertemporal budget constraint, indicating the intercepts and the slope.

   Budget constraint:

$$c_0(1+i) + c_1 = I_0(1+i) + I_1$$

$$\rightarrow 1.25c_0 + c_1 = 125$$

   Horizontal intercept: $c_0 = 100 - \frac{c_1}{1.25}$ → Horizontal intercept: 100
   Vertical intercept: $c_1 = 125 - 1.25c_0$ → Vertical intercept: 125
   Slope:-1.25

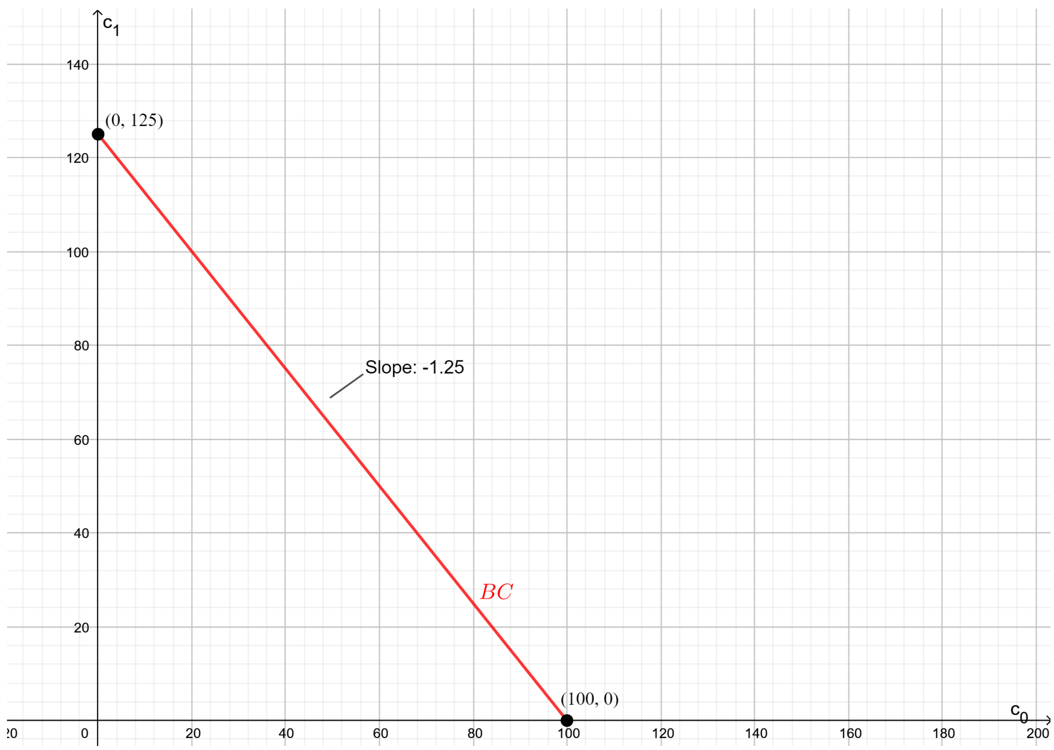
2. Provide the economic interpretation of the intercepts and the slope.

   horizontal intercept→ the amount that Bill can consume today if he borrows the amount I_1 on market interest rate. That means, that Bill only consumes in the first period.

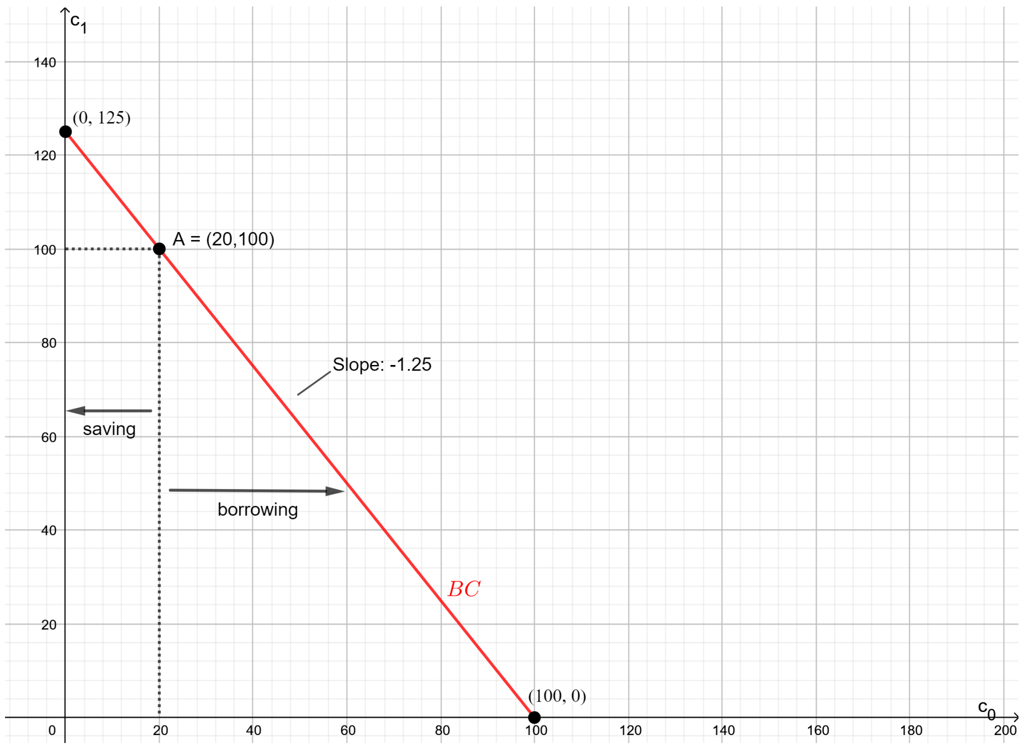
   vertical intercept → the amount of income that Bill can consume in the second period when he saves all his income in the first.(he invests the whole income I_0 at the market rate in this scenario). That means, that Bill will only consume in the second period.

slope → The slope explains the trade-off that Bill has to face between the current and future consumption. It shows, how much of C_1, Bill has to give up in order to obtain an additional unit of C_0.

3. On a properly labeled graph, draw Bill's intertemporal budget constraint. Show the intercepts and the slope.



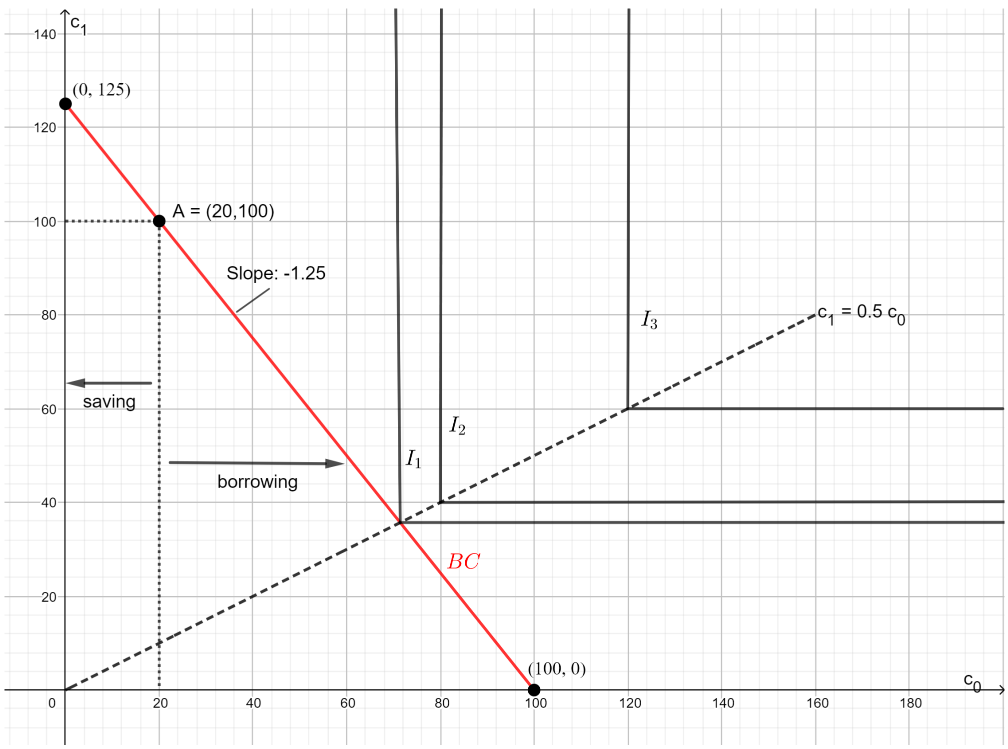
4. On the graph, show Bill's consumption plans that involve savings S and borrowing B, respectively.



5. Assume Bill's utility function is as follows:

$$U(c_0, c_1) = min(c_0, 2c_1)$$

On the graph above, draw three representative indifference curves.



6. Derive analytically Bill's optimal bundle $E^* = (c_0^*, c_1^*)$.

$$\begin{cases} c_0 = 2c_1 \\ 1.25c_0 + c_1 = 125 \end{cases}$$

$$3.5c_1 = 125 \rightarrow c_1 = 35.71$$

$$c_0 = 2c_1 = 71.43$$

$$E^* = (c_0^*, c_1^*) = (71.43, 35.71)$$

7. Derive analytically Bill's savings S_0^* or borrowing B_0^* at the optimal bundle.

   $I_0 - c_0 = 20 - 71.43 = -51.43$
   $B_0^* = 51.43 \rightarrow$ Bill borrows \$51.43 in the first period.

8. Will Bill smooth his consumption over time at the optimal bundle? (consumption smoothing means that Bill allocates his consumption in a uniform way across the two periods).

   No, he isn't smoothing the consumption because his consumption is not uniform across the two periods$\rightarrow c_0 \neq c_1$. This is shown by his utility function; therefore, c_0 is double c_1.

9. Bill and Paul succeed in arranging a meeting with the President of a well-known American electronics company. They fly to the headquarters of the company, pretty excited to present their project on which they have been working very hard. It turns out that after Bill and Paul's presentation, the President is very impressed by their work, so that he decides to fund their innovative project. The new nominal interest rate is $i = 10\%$. Derive analytically Bill's new budget constraint.

$$c_0(1+i) + c_1 = I_0(1+i) + I_1$$

$$\rightarrow 1.1c_0 + c_1 = 122$$

10. Derive analytically Bill's new optimal bundle $E^{**} = (c_0^{**}, c_1^{**})$.

$$\begin{cases} c_0 = 2c_1 \\ 1.1c_0 + c_1 = 122 \end{cases}$$

$$3.2c_1 = 122 \rightarrow c_1 = 38.13$$

$$c_0 = 2c_1 = 76.25$$

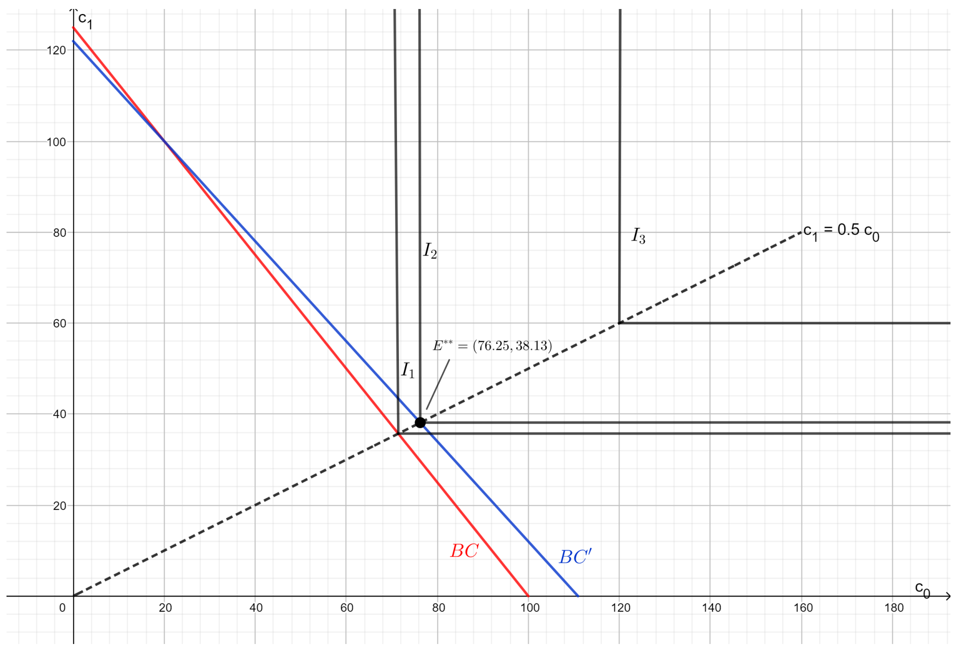
$$E^{**} = (c_0^{**}, c_1^{**}) = (76.25, 38.13)$$

11. Derive the level of savings S_1^{**} or borrowing B_1^{**} at the optimal bundle.

    The question asks for the borrowing or saving in the second period:
    $I_1 - c_1 = 100 - 38.13 = 61.87$
    $S_1^{**} = 61.87 \rightarrow$ Bill saves (\$61.87) in the second period.

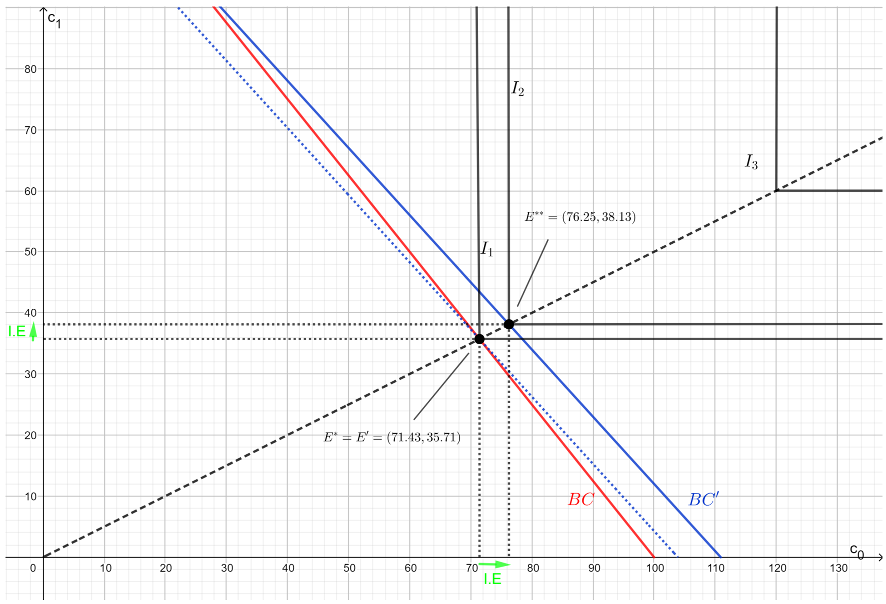
Little remark: In the first period, Bill is a borrower: $I_0 - c_0 = 20 - 76.25 = -56.25$ $B_0^{**} = 56.25 \rightarrow$ Bill borrows (\$56.25) in the first period.

12. On a properly labeled graph, show the new optimal bundle.



13. On the graph (no calculations needed), show the income and substitution effects when the nominal interest rate i decreases from 25% to 10%. Explain the two effects in detail.

There is no substitution effect, only income effect. We can see this by drawing a fictive budget constraint with the new interest rate that is tangent to the old indifference curve. We see that the new equilibrium point E' is equal to E^*. Thus the whole effect is due to the income effect (called I.E in the graph). Since Bill is a borrower, the decrease in interest rates makes him richer. Since he increases both current and future consumption when income increases, we can conclude that c_0 and c_1 are normal goods.



14. After discussing with his friend Paul, Bill's preferences over consumption plans change. Now Bill is indifferent between consuming one unit today and 1.2 units in the future. Derive analytically Bill's new utility function $U(c_0, c_1)$.

    $U(c_0, c_1) \rightarrow$ new utility function.
    Since Bill is indifferent, his utility function changes. The goods c_0 and c_1 are not perfect complements anymore but they are **perfect substitutes**.

    For perfect substitutes, the general expression for the utility function can be written in the following way:

    $$U(x1, x2) = ic_0 + jc_1$$

    One possible bundle for Bill is for example $1c_0$ and $1.2c_1$. Therefore, we know:

    $$1i = 1.2j$$

    This equality must hold for any i and j; for example, if we choose $j = 1$, we then obtain that i must be equal to 1.2. Therefore, our utility function is:

    $$U(c_0, c_1) = 1.2c_0 + c_1$$

15. Derive analytically Bill's new optimal bundles $E^{***} = (c_0^{***}, c_1^{***})$ and $E^{****} = (c_0^{****}, c_1^{****})$ when the interest rate i is 25% and 10%, respectively.

When the interest rate is 25%:

$$|MRS| = 1.2 < 1.25 = |MRT| \rightarrow |MRS \neq MRT|$$

Since $|MRS| \neq |MRT|$, we obtain a **corner solution**. We also see, $|MRS| < |MRT|$. This means, that the market asks Bill to give up more units of c_1 for one additional unit of c_0 than Bill actually is willing to give up. This means, that Bill will only consume in the second period c_1. Therefore:

$$E^{***} = (0, 125)$$

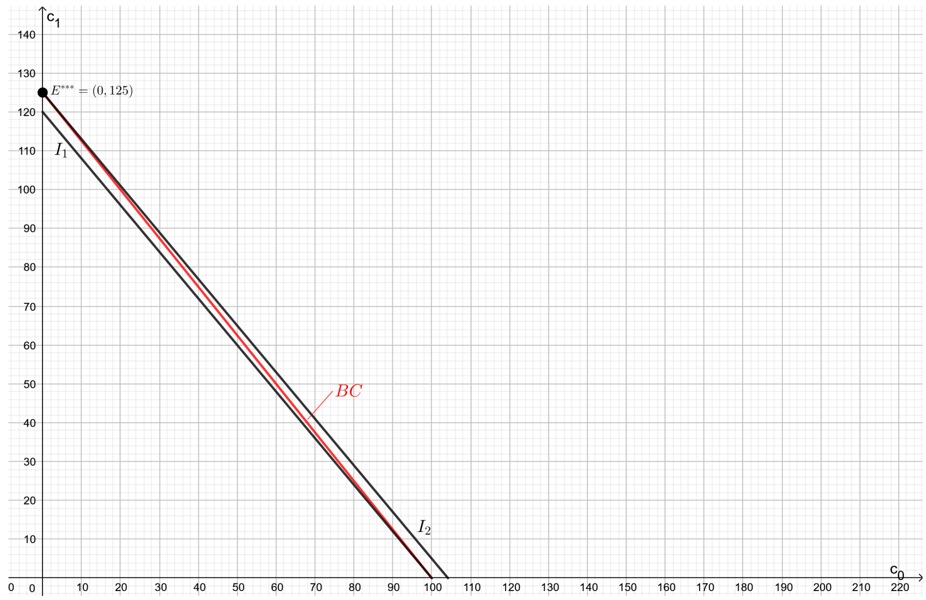
When the interest rate is 10%:

$$MRS = 1.2 > 1.1 = MRT \rightarrow MRS \neq MRT$$

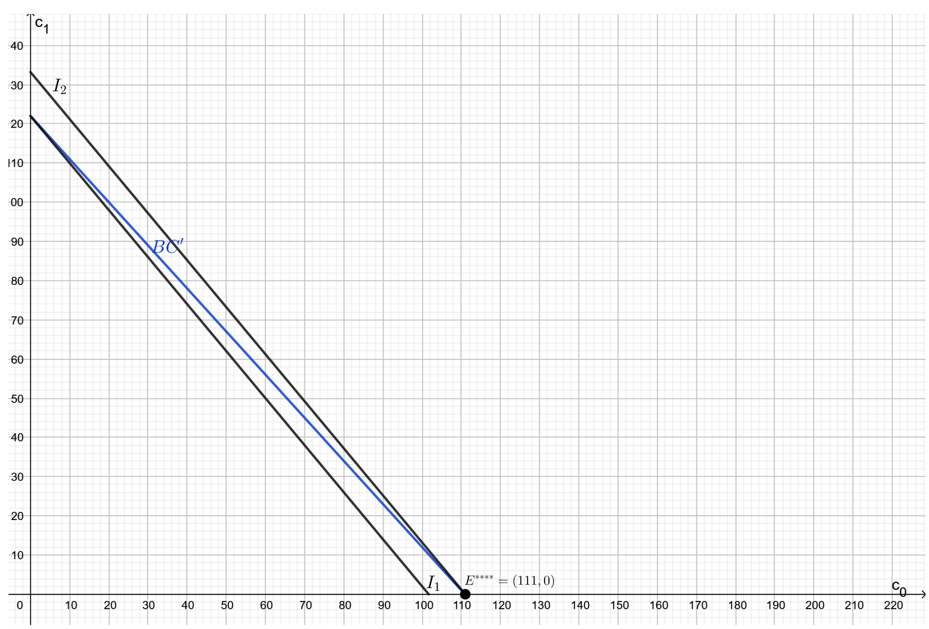
Again, we obtain a corner solution. But, this time $|MRS| > |MRT|$. So, Bill will only consume in the first period c_0.

$$E^{****} = (111, 0)$$

16. Show these two optimal bundles on two properly labeled graphs (with indifference curves and budget constraints).
The following graph shows $E^{***} = (0, 125)$



The following graph shows $E^{****} = (111, 0)$:



17. What are the consequences of the change in Bill's preferences with respect to his optimal bundles and his saving or borrowing decisions? Explain your answer.

    The change in preferences causes that Bill will consume only in either period one ($i = 10\%$) or only in period two($i = 25\%$).This means, that in the case with $i = 10\%$, Bill will borrow the present value of his future income ($\frac{I_1}{1+i} = \frac{100}{1.1}$). In the second case, when $i = 25\%$, Bill will save his entire income I_0 and invest it at the market rate $i = 25\%$.