Mathematics (BEMACS) - A. Y. 2019/20

Exercises - Set 9

1.1 Find the equation of the tangent line to the graph of the function $f : A \subseteq \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \frac{1}{x-1} - \frac{1}{(2x+1)^3}$$

at $x_0 = -1$.

1.2 Consider the function $f : A \subseteq \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = (x-1)^2 e^{2x-4}$$

(a) Calculate the derivative of f.

(b) Find the equation of the tangent line to the graph of f at $x = 2$.

1.3 Find the differential of the function $f : A \subseteq \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = (x-1)^2 - \frac{3}{x+1}$$

at $x_0 = 2$.

1.4 Find the first differential of the function $f : A \subseteq \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \sqrt{3x-2} - \left(\frac{1}{x-1}\right)^2$$

at $x_0 = 2$.

1.5 Find the second derivatives of the following function $f : \mathbb{R} \to \mathbb{R}$ defined by:

(a) $f(x) = \sqrt{1+2x}$ (b) $f(x) = \ln(1+2x)$ (c) $f(x) = (x+1)\, e^{3x}$

1.6 Find the second derivatives of the following function $f : \mathbb{R} \to \mathbb{R}$ defined by:

(a) $f(x) = x \ln(x-1)$ (b) $f(x) = \ln\left(x^2+1\right)$ (c) $f(x) = \dfrac{x}{x^2+1}$

1.1 The equation of the tangent line is $y = f(-1) + f'(-1) + (x+1)$. The derivative of $f(x) = (x-1)^{-1} - (2x+1)^{-3}$ is

$$f'(x) = (-1)(x-1)^{-2} - (-3)(2x+1)^{-4}\, 2 = -\frac{1}{(x-1)^2} + \frac{6}{(2x+1)^4}$$

From $f(-1) = 1/2$ and $f'(-1) = 23/4$ we have $y = \frac{23}{4}x + \frac{25}{4}$.

1.2 **(a)** The derivative of the function is

$$f'(x) = 2(x-1)e^{2x-4} + (x-1)^2 e^{2x-4} 2 = 2e^{2x-4}(x-1)[1+(x-1)] = 2x(x-1)e^{2x-4}$$

and $f'(2) = 4$; **(b)** The equation of the tangent line to the graph of the function at the point $x = 2$ is

$$y - f(2) = f'(2)(x-2) \qquad \Rightarrow \qquad y = 4x - 7$$

1.3 $\mathrm{d}f(x) = \frac{7}{3}(x-2)$

1.4 $\mathrm{d}f(x) = \frac{11}{4}(x-2)$

1.5 **(a)** f is defined on $[-1/2, +\infty)$. The derivative of f is

$$f'(x) = \left(\sqrt{1+2x}\right)' = \left[(1+2x)^{1/2}\right]' = \frac{1}{2}(1+2x)^{-1/2} 2 = \frac{1}{\sqrt{1+2x}}$$

and we observe that f is differentiable on $(-1/2, +\infty)$. The second derivative of f is

$$f''(x) = \left[(1+2x)^{-1/2}\right]' = -\frac{1}{2}(1+2x)^{-3/2} 2 = -\frac{1}{\sqrt{(1+2x)^3}}$$

(b) $-4(2x+1)^2$; **(c)** $e^{3x}(9x+15)$.

1.6 **(a)** $\frac{x-2}{(x-1)^2}$; **(b)** $\frac{2(1-x^2)}{(x^2+1)^2}$; **(c)** $\frac{2x(x^2-3)}{(x^2+1)^3}$.