30409 Macroeconomics (BEMACS)

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The IS-LM Model with Expectations

Luigi Iovino and Marco Maffezzoli

Bocconi University

1 Introduction

In these notes we will derive the IS-LM model starting from basic microeconomic concepts. This process is called "microfoundation" and serves two purposes. First, it makes sure that the models we use to address in macroeconomics are internally coherent. We will in fact derive the IS-LM model starting from well-understood concepts, such as utility maximization and market equilibrium. Second, by microfounding the main parts of the textbook IS-LM model, we will obtain richer implications from it. For example, *expectations* of future economic variables and household welfare (utility), which were neglected in the standard IS-LM model, will now emerge naturally.

2 A two-period model (Consumption)

The representative agent in the economy lives for two periods. She receives net income $Y - T$ and $Y' - T'$ in period 1 and period 2, respectively, and chooses consumption C and C' in period 1 and period 2, respectively, so as to maximize

$$U = \max_{C,C'} \log C + \beta \log C'$$

subject to the intertemporal budget constraint (BC)

$$pC + \frac{p'C'}{1+i} = p\left(Y - T\right) + \frac{p'\left(Y' - T'\right)}{1+i}.$$

Here p and p' denote, respectively, the price of the period-1 good and the price of the period-2 good and i is the (nominal) interest rate, that is, the interest rate on a bond that costs €1 in period 1 and pays off €$(1+i)$ in period 2.

From the definition of inflation rate

$$\pi = \frac{p'}{p} - 1,$$

thus, we can rewrite the BC as follows:

$$C+\frac{C'}{\frac{1+i}{1+\pi}}=Y-T+\frac{Y'-T'}{\frac{1+i}{1+\pi}}. \tag{1}$$

Note that the ratio $\frac{1+i}{1+\pi}$ is actually the *real* interest rate: It tells you how many units of future consumption you can obtain, if you give up one unit of current consumption. We can therefore, define:

$$1+r \quad \equiv \quad \frac{1+i}{1+\pi}.$$

To solve the agent's maximization problem, we use (1) to solve for C and then replace it in the utility function. The problem becomes

$$U \quad = \quad \max_{C'} \log\left(Y-T+\frac{Y'-T'}{1+r}-\frac{C'}{1+r}\right)+\beta\log C'.$$

This is a concave problem in one variable. Therefore, the following first-order condition (FOC) is necessary and sufficient for the solution:

$$\underbrace{\frac{1}{Y-T+\frac{Y'-T'}{1+r}-\frac{C'}{1+r}}}_{=C}\left(-\frac{1}{1+r}\right)+\beta\frac{1}{C'}=0,$$

which can be rewritten as

$$\frac{1}{C} \quad = \quad (1+r)\,\beta\frac{1}{C'}, \tag{2}$$

$$\tag{3}$$

Equation (2) is known as the *Euler equation.* The interpretation is simple: If the agent decides to give up one unit of current consumption and save it, she will lose $\frac{1}{C}$ in terms of utility (this is marginal utility in period 1). By saving this unit, however, the agent obtains $1+r$ units of consumption in the future, which results in a gain of $\beta\frac{1}{C'}$ in terms of future utility (this is marginal utility in period 2). The Euler equation says that, at the optimum, the agent must be indifferent between consuming the unit in the present or saving it for the future. If this was not the case, in fact, the agent could increase her utility by either consuming more in the present or by consuming more in the future, which would contradict optimality.

Finally, we can combine the Euler equation (2) and the BC to get a nice expression for current consumption C (a similar expression can be derived for future consumption C'):

$$C=\underbrace{\frac{1}{1+\beta}}_{\equiv c_1}(Y-T)+\underbrace{\frac{1}{1+\beta}\frac{1}{1+r}}_{\equiv c_0}(Y'-T').$$

This expression deserves a few comments. First, if we collect $\frac{1}{1+\beta}$, we can rewrite C as

$$C=\frac{1}{1+\beta}\left[(Y-T)+\frac{1}{1+r}(Y'-T')\right].$$

The term in square brackets is the present discounted value (PDV) of future (net) income. This is known as *human capital*. Therefore, the agent consumes a constant fraction of her human capital.

Second, note that if we define $c_1 \equiv \frac{1}{1+\beta}$ and $c_0 \equiv \frac{1}{1+\beta}\frac{1}{1+r}(Y' - T')$, then we have the standard consumption function assumed in the IS-LM model:

$$C = c_0 + c_1 (Y - T).$$

Thus, utility maximization can be used to justify one of the main assumptions of the IS-LM model. What is more, by deriving C from first principles (utility maximization), we have learnt that the constant c_0 depends on future variables and on the interest rate. Therefore, we can augment the standard IS-LM model by incorporating expectations of future variables. In particular, we can write

$$C\left(\underset{+}{Y - T}, \underset{+}{Y' - T'}, \underset{-}{r}\right), \tag{4}$$

where the signs below the variables indicate how each variable affects consumption.

3 A two-period model (Investment)

We now consider the problem of the typical firm in the economy. As for the consumer, we assume that also the firm lives for two periods. The firm maximizes the PDV of profits:

$$\Pi + \frac{\Pi'}{1+r},$$

where Π and Π' denote, respectively, current and future profits. In turn, profits equal revenues minus costs.

To produce output Y a firm must employ labor and capital. Formally, $Y = AK^{\alpha}N^{1-\alpha}$ and, in the future, $Y' = A'(K')^{\alpha}(N')^{1-\alpha}$. Here, A and A' capture the productivity of the firm: For given inputs (capital and labor), a more productive firm can produce more. Finally, we set the price of the good produced by the firm equal to 1.

We assume that the firm begins its life with some capital K, which it cannot change. However, the firm can change *future* capital K' by investing an amount I. Current profits are given by

$$\begin{aligned} \Pi &= \underbrace{Y}_{\text{revenues}} - \underbrace{(wN + I)}_{\text{costs}} \\ &= AK^{\alpha}N^{1-\alpha} - (wN + I). \end{aligned}$$

Furthermore, by investing I today, the firm can create future capital K' according to the following equation:

$$K' = (1-\delta)K + I. \tag{5}$$

Here, δ is the depreciation rate: After using capital K, a fraction δ of it is lost (think of this as normal "wear and tear"). Thus, only the remaining fraction $1-\delta$ can be used for production in the next period.

Remember that the firm lasts only for two periods. Thus, after producing in the last period, it will sell all the remaining capital and shut down. Formally, future profits are given by

$$\begin{aligned}\Pi' &= \underbrace{Y' + (1-\delta)\,K'}_{\text{revenues}} - \underbrace{w'N'}_{\text{costs}} \\ &= A'\,(K')^{\alpha}\,(N')^{1-\alpha} + (1-\delta)\,K' - w'N'.\end{aligned}$$

Putting things together, the firm chooses current labor, future labor, and investment as follows:

$$\max_{N,N',I} \left[AK^{\alpha}N^{1-\alpha} - (wN+I)\right] + \frac{A'\,(K')^{\alpha}\,(N')^{1-\alpha} + (1-\delta)\,K' - w'N'}{1+r}.$$

It turns out that we don't need to solve this problem fully. Instead, it is enough only to solve for the optimal choice of I.

Remember that K' varies with investment as showed by (5), so we must take that into account when we differentiate. Formally, taking the derivative of Π with respect to I gives the FOC

$$-1 + \frac{1}{1+r}\left[\alpha A'\,(K')^{\alpha-1}\,(N')^{1-\alpha} + (1-\delta)\right]\frac{\partial K'}{\partial I} = 0,$$

where $\frac{\partial K'}{\partial I}$ is the (partial) derivative of K' with respect to I. From (5), we have that

$$\frac{\partial K'}{\partial I} = 1.$$

Therefore, the FOC for the firm is

$$-1 + \frac{1}{1+r}\left[\alpha A'\frac{(N')^{1-\alpha}}{(K')^{1-\alpha}} + (1-\delta)\right] = 0.$$

If we multiply and divide the first term in the square bracket by $(K')^{\alpha}$, we have

$$-1 + \frac{1}{1+r}\left[\alpha A'\frac{(N')^{1-\alpha}\,(K')^{\alpha}}{(K')^{1-\alpha}\,(K')^{\alpha}} + (1-\delta)\right] = 0$$

or, using the fact that $Y' = A'\,(K')^{\alpha}\,(N')^{1-\alpha}$,

$$-1 + \frac{1}{1+r}\left[\alpha\frac{Y'}{K'} + (1-\delta)\right] = 0.$$

Finally, we can rewrite the latter as

$$\alpha\frac{Y'}{K'} = r + \delta.$$

This condition is easy to interpret. It says that, at the optimum, the firm will invest up to the point where the marginal product of one extra unit of capital ($\alpha\frac{Y'}{K'}$) is equal to the marginal cost of financing the capital plus the loss due to depreciation ($r+\delta$).

Using (5) we have

$$\alpha \frac{Y'}{(1-\delta)K+I} = r+\delta$$

or

$$I = \frac{\alpha Y'}{r+\delta} - (1-\delta)K.$$

Note that investment increases with future output Y' and decreases with the interest rate (r). It also decreases with current capital K (if the firm already has capital, it needs to invest less for the future). We can thus write

$$I\left(\underset{+}{Y'}, \underset{-}{r}\right). \tag{6}$$

This is almost identical to the investment function in the standard IS-LM model. However, investment now depends on *future* output, not the current one. This is very intuitive: The firm invests to produce more in the future, thus, it cares about future production/income, not current one.

4 IS-LM with expectations

We can now use the new consumption function (4) and the new investment function (6) to incorporate expectations into our old IS-LM model. Before doing that, we can also add the risk-premium into our formulas. The quickest and easiest way to do this is to assume that, since the environment is risky and investors are risk-averse, they require an extra compensation x (i.e., a risk premium) to invest in risky activities. Basically, this means that we simply need to add x to the interest rate r in (4) and (6).

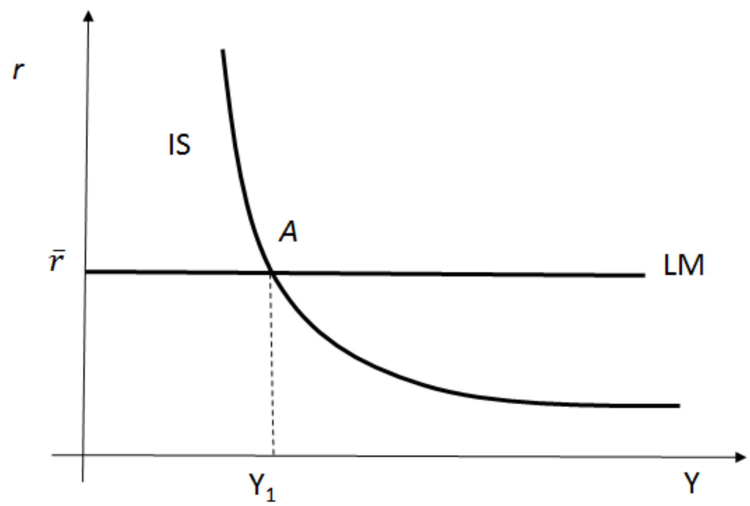
We are now ready for the new IS-LM model. We assume that the central bank has still control over the real interest rate, hence, the LM curve is still

$$r = \bar{r}.$$

Instead, the IS curve is now

$$Y = C\left(Y-T, Y'-T', r+x\right) + I\left(Y', r+x\right) + G.$$

Since Y and r have similar effects on consumption and investment as in the the old IS-LM model, the graphical representation of the model is basically the same.



Finally, as the next example shows, working with the new IS-LM is very simple.

Example. Suppose agents expect *future* income to increase. What happens to *current* income? Since future income has a positive effect both on consumption and investment, we should expect the IS to shift rightward. In turn, this causes an increase in current income Y.